TOWARDS STABLE ADAPTIVE CONTROL FOR NONLINEAR SYSTEMS

BY

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TABLE OF CONTENTS

		Page
ACKNOWLEDGEMENTS		ii
ABSTRACT		v
CHAPTERS		
I	INTRODUCTION	1
II	A METHOD FOR INCLUDING ROBUSTNESS TO ADAPTIVE CONTROLLERS	5
	Introduction	5
	The Self-Tuning Regulator	6
	The Overall Control Scheme	9
	A Stabilizing Controller	12
	Example	15
III	A CAUTIOUS SELF-TUNING CONTROLLER	21
	Introduction	21
	The Cautious Self-Tuning Controller Structure	22
	The "Caution" Term	26
	Example	28
IV	A MIMO CAUTIOUS SELF-TUNING CONTROLLER	45
	Introduction	45
	Model	45
	Multivariable Cautious Self-Tuning Controller	51
	Stability and Robustness of CSTC	55
V	MODEL DEPENDENT CONTROL ANALYSIS	57
	Introduction	57
	The MDC Method	58
	Closed-Loop Polynomial Selection	63
	MDC Algorithm	66
	Robustness of MDC Law	68
VI	ADAPTIVE MODEL DEPENDENT CONTROL	77
	Introduction	77
	Adaptive MDC Law	79
	Cautious Adaptive MDC Tuning	89
	Stability and Convergence Analysis of Adaptive MDC and Cautious Adaptive MDC	91
	Example	102

Page

VII ALGORITHM FOR ADAPTIVE CONTROL OF ACTIVATED SLUDGE PROCESSES 113
Introduction 113
Process Description-Model for Simulation 116
Control Objectives and Algorithms 122
Simulation Results 131

VIII SUMMARY AND CONCLUSIONS 147

APPENDICES

A INTERNAL MODEL CONTROL 150

B RECURSIVE LEAST SQUARES ESTIMATOR 162

C PROPERTIES OF MDC POLE PLACEMENT ALGORITHM 174

D PROPERTIES OF TIME VARYING LINEAR DISCRETE OPERATORS 178

REFERENCES 194

BIOGRAPHICAL SKETCH 200

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The problem of robustness of adaptive controllers in the presence of nonlinear and/or unmodeled dynamics is addressed. The performance and robustness of the recently introduced Model Dependent Control (MDC) are examined for the non-adaptive case. Adaptive MDC is presented and the stability for application to linear systems is investigated. Finally, a novel adaptive controller is presented for application to activated sludge processes.

The stabilizing and the Cautious Self-Tuning Controller are introduced. These controllers are able to perform effectively in the presence of nonlinearities and model uncertainty typical of industrial processes. They incorporate a dead-beat adaptive controller and a robust one independent of the estimated model. The robust controller becomes dominant in case of a large plant/model mismatch, switching to the adaptive controller as the estimated model improves. Both controllers are applicable to open-loop stable and minimum phase systems.

Simulations with highly nonlinear models demonstrate their effectiveness, even when other adaptive controllers give unsatisfactory performance.

The non-adaptive MDC method is revisited so as to handle independently set-point tracking and disturbance rejection. Only two parameters need to be tuned and these influence directly control quality and robustness. It is proved that given uncertainty bounds a robust MDC controller can be designed even for open-loop unstable systems.

Two adaptive controllers based on the MDC methodology are presented. For the first, the MDC tuning parameters are constant, whereas for the second they are tuned on-line depending on the estimated model error. Stability is established for both controllers for application to linear time invariant systems. The adaptive MDC is applicable to open-loop unstable and/or minimum phase systems.

An adaptive optimal bang-bang control algorithm is developed that compensates for diurnal feed variations in the activated sludge process for wastewater treatment. Also, the Cautious Self-Tuning Controller is employed for regulation of the dissolved oxygen concentration. Simulations with an activated sludge model are carried out and the performance of both controllers is studied.

CHAPTER I
INTRODUCTION

Process control is concerned with the manipulation of some of the process inputs so that the process outputs achieve certain objectives. The most common objectives in the chemical industry are set-point following and regulation. In designing a controller the first step is to obtain a mathematical model which describes the dynamic relationship between the process inputs and outputs. Having a model there is a large number of design methods to generate control laws. Each of these methods is based on a specific performance objective for the process. Most of the controller design methods are based on linear models. This is due to the fact that linear models are easily obtained and that linear differential or difference equations admit analytic solutions which then allow the study of the closed-loop process performance.

Adaptive control was developed for cases where the model has unknown parameters or the parameters change with the operating conditions. The large majority of adaptive control algorithms are based on the following principles: the parameters of a linear (or bilinear) model are identified on-line using a parameter estimator and the control law is calculated from the current parameter estimates as if they were the true ones. This approach is called "certainty equivalence adaptive control." If, in addition, the parameter estimation error is used to tune the controller, the approach is called "cautious adaptive control."

Adaptive controllers feature an important advantage over conventional controllers. They are able to tune themselves to optimal

settings and to retune when a change occurs in the process parameters. This is where the best known adaptive controller, the Self-Tuning Regulator (STR), draws its name from. The STR presented in its final form by Astrom and Wittenmark (1973) utilizes a minimum variance controller design method and it is appropriate for regulating stochastic systems. Clarke and Gawthrop (1975) presented an extension to the STR, the Self-Tuning Controller (STC), which incorporates input penalization in the control objective. Other important adaptive schemes are Model Reference Adaptive Control (MRAC) (Landau, 1981), pole-zero placement controllers (e.g. Wellstead et al., 1979; Astrom and Wittenmark, 1980) and bilinear STCs (e.g. Svoronos et al., 1981).

Adaptive controllers are generally based on on-line identified linear models. The stability and convergence to the target steady state of several adaptive controllers, including the minimum variance STR when applied to linear systems, were recently established (e.g. Goodwin et al., 1984). However, most chemical processes are inherently nonlinear due to nonlinear dependence of reaction rates and physical properties on temperature, concentration, pressure, and geometry. Other nonlinearities are added by nonlinear sensors and control valves as well as by controller saturation. The existing linear-model-based adaptive algorithms face several problems when applied to highly nonlinear systems. For instance, the minimum variance STR may lead to poor behavior due to plant/model mismatch during initialization and large set point or load changes. One might very well observe large sustained oscillations or even failure of the algorithm to drive the system to the desired set point. Such incidents have been reported by Song et al. (1984) and Gustafsson (1984). An adaptive controller based on the

minimization of a quadratic index which includes penalty on control deviation from the steady-state value (i.e. Clarke and Gawthrop, 1975) can reduce oscillations, but requires accurate knowledge of the steady-state control values; otherwise it results in offset. In addition, a high value of penalty results in sluggish response, which is unnecessary if the on-line identified model is good. This last problem is shared by STCs based on the minimization of a quadratic index which includes penalty in the control velocity (e.g. Clarke and Gawthrop, 1975). Velocity penalization may actually result in increased oscillations as well. Pole/zero placement adaptive controllers with fixed pole specifications are faced with similar problems. An alternative for some nonlinear systems is the use of nonlinear adaptive controllers. Promising algorithms have been proposed by Golden and Ydstie (1985) and by Gustafsson (1984), but these require that the nonlinearities be known a priori and be static, conditions seldom fulfilled in practice.

The main objective of this work is to develop simple and robust adaptive controllers which perform satisfactorily during both regulation and set-point changes when applied to systems which deviate significantly from the ideal linear behavior.

In Chapters II-IV modifications of the STC design are presented which enhance the robustness characteristics of conventional STCs. Chapter II describes a control scheme which consists of two parts. An adaptive controller which regulates the system close to the operating steady state and a robust non-adaptive controller which takes over when the model is poor or whenever a large set-point or load change occurs. The algorithm switches from one controller to the other depending on the magnitude of the estimation and tracking errors.

In Chapter III the same idea is refined so that the transition between the two controllers is smooth. The algorithm retains the structure and simplicity of STC. Since the estimation error is involved in obtaining the control law, the presented adaptive controller is called the Cautious STC (CSTC). In Chapter IV the CSTC is extended to multi-input multi-output (MIMO) systems.

The controllers presented in Chapters II-IV can be applied only to open-loop stable and minimum phase systems with constant a priori known time delays. Non-minimum phase systems can be treated but this requires extensive process knowledge. In Chapter V a novel controller design methodology is presented, namely the Model Dependent Control (MDC) which can handle open-loop unstable and non-minimum phase systems with unknown or perhaps slowly varying time delays. The non-adaptive MDC was introduced by Svoronos (1986). In Chapter V an improved version is derived which is able to handle independently set-point tracking and disturbance rejection. The controller robustness is also studied in the case of model uncertainty. In Chapter VI a certainty equivalence and a cautious adaptive controller based on the MDC methodology are presented. The asymptotic stability and convergence of both controllers when applied to linear deterministic systems are studied. Finally, in Chapter VII an adaptive optimal bang-bang controller and a cautious bilinear STC are presented. Both controllers are specifically designed for application to the activated sludge process for wastewater treatment.

CHAPTER II
A METHOD FOR INCLUDING ROBUSTNESS TO ADAPTIVE CONTROLLERS

Introduction

As mentioned earlier if a linear model based adaptive controller is applied to a process with significant nonlinearities, it might fail. The reason of failure becomes evident through the following arguments: If the parameter estimates are poor (as is often the case after initialization) then the adaptive control policy which is based on these poor estimates may generate an erroneous signal that will bring the system away from the desired operating state. Alternatively, a large disturbance or set point change may drive the system away from the desired steady state. Once away from it, the nonlinear terms become significant. Since the parameter estimator attempts to fit a local linear model to the process, it is possible that the parameter estimates will fail to converge to reasonable values. Erroneous parameter estimates lead to erroneous control action, continual application of which may even lead to instability.

Thus, adaptive control schemes which are based on local models generally have only a limited region of stability around the desired steady state. In this chapter a method is presented for modifying Single Input Single Output (SISO) adaptive control algorithms, which can greatly increase this region of stability.

In the development that follows the adaptive algorithm is the STR. Exactly the same development can be followed with other linear model based adaptive controllers that use different performance objectives such as the Clarke-Gawthrop STC, pole/zero placement algorithms or bilinear STCs.

The Self-Tuning Regulator

The well known minimum variance STR of Astrom and Wittenmark (1973) assumes that the process can be described by the discrete-time SISO model

$$y(t+k+1) = A(q^{-1})\, y(t+k) + B(q^{-1})\, u(t) + s + \zeta'(t+k+1) \qquad (2\text{-}1)$$

$$A(q^{-1}) = a_0 + a_1\, q^{-1} + \ldots + a_n q^{-n} \qquad (2\text{-}2)$$

$$B(q^{-1}) = b_0 + b_1\, q^{-1} + \ldots + b_m q^{-m} \qquad (2\text{-}3)$$

where t denotes time in number of sampling intervals; y and u are the sampled system output and the control variable, respectively; $\zeta'(t+k+1)$ is a zero mean noise sequence uncorrelated to previous inputs and outputs; $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1} ($q^{-1}\, f(t) = f(t-1)$); s is a load parameter; and k is the system dead time in number of sampling intervals. The model parameters a_i, b_i, s are assumed unknown and may be slowly time-varying. It is, however, required that the deadtime k and upper bounds to the system orders n and m be known a priori.

The minimum variance STR attempts to minimize the performance measure

$$J(u(t)) = E\{[y(t+k+1) - y_d]^2 | t\} \qquad \text{(stochastic case)}$$
$$= [y(t+k+1) - y_d]^2 \qquad \text{(deterministic case)} \tag{2-4}$$

where y_d is the desired set point and E is the expectation operator. To solve the minimization problem the predictor form of (2-1) is first obtained by sequentially substituting for y(t+k), y(t+k+1), ..., y(t+1) using (2-1) shifted by the appropriate number of sampling intervals

$$y(t+k+1) = C(q^{-1})\, y(t) + D(q^{-1})\, u(t) + r + \zeta(t+k+1) \tag{2-5}$$

where

$$C(q^{-1}) = c_0 + c_1\, q^{-1} + \ldots + c_n\, q^{-n} \tag{2-6}$$

$$D(q^{-1}) = d_0 + d_1\, q^{-1} + \ldots + d_{m+k}\, q^{-(m+k)}\ ;\quad d_0 = b_0 \tag{2-7}$$

and $\zeta(t+k+1)$ is a linear combination of $\zeta(t+1), \ldots, \zeta(t+k+1)$. In vector notation

$$y(t+k+1) = \theta^T x(t) + \zeta(t+k+1) \tag{2-8}$$
$$= \theta_1^T x_1(t) + d_0\, u(t) + \zeta(t+k+1)$$

where

$$\theta_1^T = [c_0, c_1, \ldots, c_n, d_1, d_2, \ldots, d_{m+k}, r] \tag{2-9}$$

$$x_1^T(t) = [y(t), \ldots, y(t-n), u(t-1), \ldots, u(t-m-k), 1] \tag{2-10}$$

$$\theta^T = [\theta_1^T, d_0] \tag{2-11}$$

$$x^T(t) = [x_1^T(t), u(t)] \tag{2-12}$$

Substitution of (2-8) into (2-4) gives the minimum variance (MV) control law

$$u^*_{MV}(t) = \frac{1}{d_0} [y_d - x_1(t)\ \theta_1) \tag{2-13}$$

This achieves $y(t+k+1) = y_d$ (or $E(y(t+k+1)) = y_d$) and, from a pole placement point of view, cancels some of the closed-loop poles with the open-loop zeroes and sets the rest of them to the origin. Thus $u^*_{MV}(t)$ is a controller that maximizes speed of response.

For the STR the unknown parameter vector is estimated via a recursive estimator and the current estimates $\hat{\theta}(t)$ are used in (2-13) in the place of the true parameters. The implemented control law is

$$u_{MV}(t) = \frac{1}{\hat{d}_0(t)} [y_d - x_1^T(t)\ \hat{\theta}_1(t)] \tag{2-14}$$

where the superscript $\hat{}$ denotes parameter estimate.

The above described MV STR has been proved recently to be globally stable when applied to linear systems (Ydstie and Sargent, 1982; Cordero and Mayne, 1981; Goodwin et al., 1980). It is a superior controller for linear or almost linear processes but is unreliable for highly nonlinear ones and can lead to failures of various types. As mentioned previously these failures range from unacceptably oscillatory regulation around the set point to instability. They are due to the high sensitivity of the MV control law on the estimated model error which can be large when the nonlinearities become dominant.

It is noted again that the STR of (2-14) is used as an example and in fact any other adaptive controller could be used in its place.

The Overall Control Scheme

Suppose that somehow a robust controller has been found that, although it does not perform as well as the STR in the neighborhood of the steady state (where the system is most of the time), it is able to drive the system towards the desired steady state after large deviations which the STC cannot handle. Let this controller be referred to as the "stabilizing" controller $u_s(t)$. Then it is reasonable to control the system close to the steady state with the STR and far away form it with the "stabilizing" controller. To be more specific, we say that the system is close to the steady state if both the measurement error

$$\varepsilon_1(t) \equiv |y(t) - y_d|$$

and the prediction error

$$\varepsilon_2(t) \equiv |y(t) - \hat{y}(t|t-k-1)|$$

satisfy

$$\varepsilon_1(t) \leq \varepsilon_{1max}$$

$$\varepsilon_2(t) \leq \varepsilon_{2max}$$

where ε_{1max} and ε_{2max} are chosen to be small fractions of $|y_d|$ (if $|y_d| \neq 0$) and can be taken equal (note that for a linear process $\varepsilon_1(t) = \varepsilon_2(t)$).

The overall control scheme which is shown schematically in Figure 2-1 is as follows:

If either $\varepsilon_1 \geq \varepsilon_{1max}$ or $\varepsilon_2 \geq \varepsilon_{2max}$, the process is controlled via $u_S(t)$ but the STR parameters $\hat{\theta}$ continue to be updated.

If both errors are within their limits, the process is controlled by a velocity bounded STC

$$u(t) = \begin{cases} u(t-1) + \Delta u_{max} & \text{if } u_{MV}(t) > u(t-1) + \Delta u_{max} \\ u(t-1) - \Delta u_{max} & \text{if } u_{MV}(t) < u(t-1) - \Delta u_{max} \\ u_{MV}(t) & \text{otherwise} \end{cases}$$

The bound Δu_{max} can be absolute or relative ($\Delta u_{max} = c|u(t-1)|$) and is necessary for the following reason. The "stabilizing" controller is a time invariant feedback control law. It has been shown in Astrom and Wittenmark (1973) and in Svoronos et al. (1981) that in case of a time invariant feedback serious identifiability problems are experienced during closed-loop operation. Thus, when the "stabilizing" controller

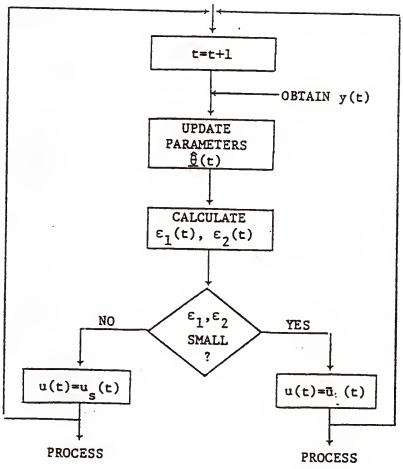


Figure 2-1. The "stabilizing" STC.

reduces ε_1, ε_2 below ε_{1max}, ε_{2max} respectively and the STC takes over, the parameter estimates might be completely inappropriate even though $|\varepsilon_2(t)|$ is small. Therefore, $u_{MV}(t)$ might be incorrect and if it were not bounded, it might throw the system far away from the desired steady state. Alternatively, since the system is not far from the target steady state (when $\bar{u}$ is used) large control velocity should not be needed. The velocity bound provides for a smooth transition between the two control laws.

From hereon the overall scheme will be referred to as the "stabilizing" STC.

A Stabilizing Controller

Controller Specifications

Suppose that the process is satisfactorily controlled by the STR around the desired steady state (y_{d1}, u_{d1}) and that the parameter estimates have converged inside a ball in the parameter space with center the true parameter vector θ'. Now assume that at some instant a load or set point change occurs and as a result the new steady state changes to (y_{d2}, u_{d2}). Since the process is nonlinear, the parameters of the locally valid linear model also change, say to θ''.

If the stabilizing controller $u_s(t)$ is to succeed in bringing the system from steady state (y_{d1}, u_{d1}) to (y_{d2}, u_{d2}), the closed-loop system must at least be locally asymptotically stable at the new steady state (a necessary but not sufficient condition). This is equivalent to the condition that system (2-1) or (2-5) is asymptotically stable for $\theta = \theta''$

and $u(t) = u_s(t)$. In addition, if $u_s(t)$ is to drive the output close enough to y_{d2}, it should eliminate steady-state offset.

A "stabilizing" controller satisfying all forementioned specifications is presented next.

Detuned IMC as "Stabilizing" Controller

Internal Model Control (IMC) (Garcia and Morari, 1982; Brosilow and Tong, 1978) provides a simple yet effective method for designing robust controllers for open-loop stable systems. A brief description of IMC is given in Appendix A.

To develop the IMC law we need an approximate, perhaps very rough, dynamic linear model. If such a model is not available one can operate the process open loop, but with excitation added to the control variable, about a steady state for a limited period of time. The estimator can then use the data collected to estimate the parameters of model (2-1). This model can be updated from time to time using trustworthy models identified during the STR operation.

Let the z-domain version of the obtained model be

$$\frac{Y(z)}{U(z)} = G(z) \qquad (2\text{-}15)$$

where $Y(z)$, $U(z)$ are the z-transformed $y(t)$, $u(t)$ respectively, and $G(z)$ is the model transfer function. The transfer function $G(z)$ is factored as

$$G(z) = G_+(z)\, G_-(z)$$

where $G_+(z)$ includes all the noninvertible elements of $G(z)$. That is, the time delay and the zeroes on or outside the unit circle. Then the internal model controller is given by

$$U_{IMC}(z) = \frac{F(z)}{G_-(z)[1-G_+(z)F(z)]} E(z) \qquad (2\text{-}16)$$

where $E(z) = Y_d(z) - Y(z)$ and $F(z)$ is a filter which is included to increase robustness and improve performance

$$F(z) = \frac{1-f}{1-fz^{-1}} F_1(z) \qquad 0 \leq f < 1 \qquad (2\text{-}17)$$

where $F_1(z)$ satisfies (see Appendix A)

$$|F_1(e^{j\omega})G_+(e^{j\omega})| = 1 \qquad 0 \leq \omega \leq \pi$$

It is shown in Appendix A that there exists f ($f_{min} \leq f < 1$) which stabilizes the closed-loop system, even for very poor process models. This is obvious from (2-16) and (2-17), since as $f \to 1$ the controller gain decreases towards zero. Thus, if the system is finite gain stable (bounded input bounded output stable) the Small Gain Theorem (Desoer and Vidyasagar, 1975, p. 40) implies that the closed-loop system is stable. It is also shown in Appendix A that IMC eliminates offset. Therefore, for finite gain stable systems an appropriately tuned IMC law meets all specifications set for the stabilizing controller.

To tune the "stabilizing" IMC uncertainty bounds $G(z)$ (see Appendix A) would be extremely helpful. If no such information is available a high value is recommended for f which results in a conservative IMC

tuning. In contrast, knowledge of the model uncertainty can be used to tune IMC as needed in order to achieve robustness without sacrificing performance. For open-loop unstable systems, IMC results in an unstable closed loop so it is not an appropriate "stabilizing" controller. A robust controller for these systems is developed in Chapter V.

Example

Consider the nonlinear system

$$y(t+1) = 0.9\ y(t) + 0.1\ u(t) + 10\ y(t)u(t)$$

It has order 1 (n=0), no dead time (k=0) and the predictor form is

$$\hat{y}(t+1|t) = \hat{s} + \hat{a}_0\ y(t) + \hat{b}_0\ u(t)$$

The STR control law is

$$u_{MV}(t) = \frac{y_d - \hat{s} - \hat{a}_0\ y(t)}{\hat{b}_0}$$

Figure 2-2 demonstrates the performance of $u_{MV}(t)$. The system was initialized around the steady state $u = y = 0$. At time 50 the set point was changed from 0 to 0.01 and it is seen that $u_{MV}(t)$ successfully brought the system at the new steady state. But when at $t = 100$ a larger set-point change occurred ($0.01 \rightarrow 0.05$), the STC failed and the system became unstable.

The overall control scheme of section 2.3 with the IMC of section 2.4 used as the "stabilizing" controller, i.e. the "stabilizing" STC,

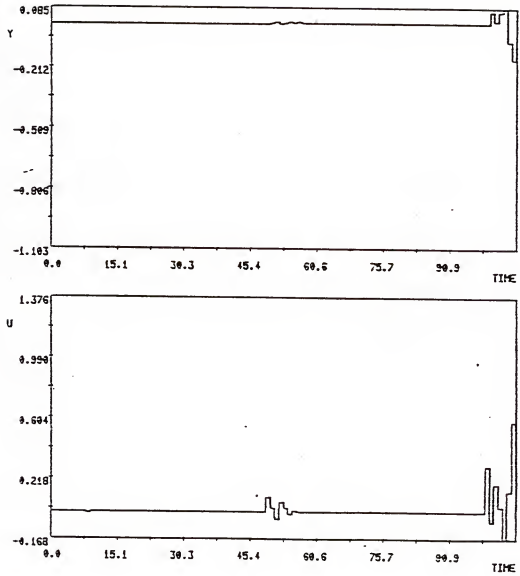


Figure 2-2. Response of the u_{MV} controlled system to set-point changes $0 \rightarrow 0.01 \rightarrow 0.05$.

performed considerably better. It succeeded for much larger set-point changes as seen from Figure 2-3 which depicts the response of the system when the second set point change was $0.01 \rightarrow 1.0$.

The control velocity bound was $\Delta u_{max} = 0.01$ and the error bounds $\varepsilon_{1max} = \varepsilon_{2max} = 0.1$. The nominal transfer function G(z) was obtained from the estimated model at the initial $u = y = 0$ steady state as

$$G(z) = \frac{\hat{b}_0}{z - \hat{a}_0}$$

where $\hat{a}_0 \simeq 0.9$ $\hat{b}_0 \simeq 0.1$. Since no information was considered available for the model uncertainy, the filter parameter was taken $f = 0.99$. The time invariant IMC law is then given by

$$\bar{U}_{IMC}(z) = 0.1 \frac{1-0.9z^{-1}}{1-z^{-1}} [Y(z) - Y_d(z)] \qquad (2\text{-}18)$$

The process was controlled with $\bar{U}(t)$ until, at time $t^* = 100$, ε_1 exceeded its limit. From time $t = 101$ to $t = 116$ (until both errors were reduced to within their limits), the process was controlled by the "stabilizing" controller of (2-18).

It is interesting to compare the response of the overall scheme to that of its components. Figure 2-4 shows the closed loop response when the bounded STC, $\bar{U}(t)$ is the controller for all ε_1, ε_2. The second set point change was again $0.01 \rightarrow 1.0$ and it is clear that the controller failed. Figure 2-5 shows how the IMC alone performs for the set-point $0.01 \rightarrow 1.0$. It is seen that it took considerably longer than the "stabilizing" STC to reach the desired steady state.

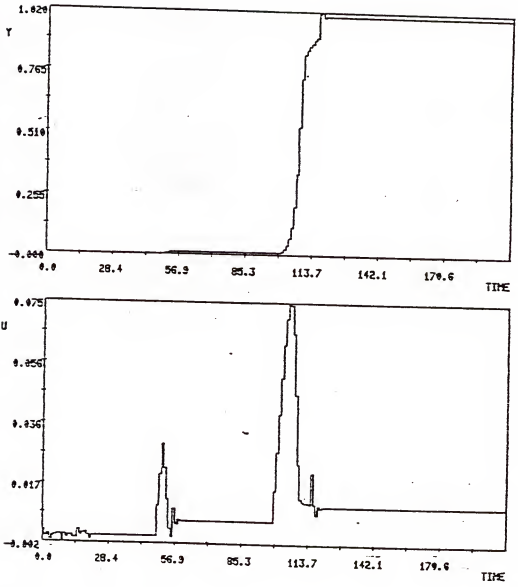


Figure 2-3. Response of the "stabilizing" STC controlled system to set-point changes $0 \rightarrow 0.01 \rightarrow 1$.

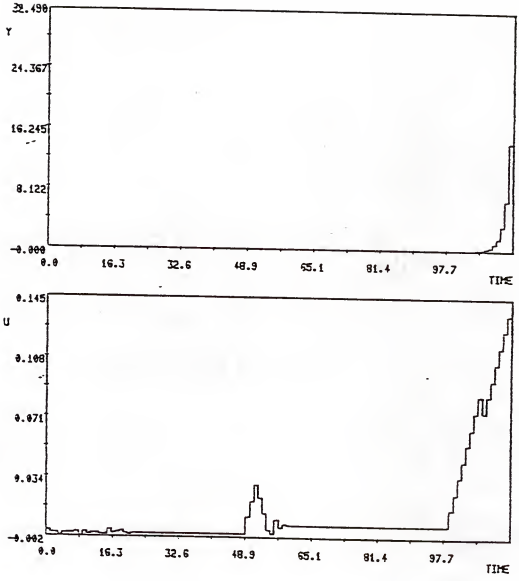


Figure 2-4. Response of the $\bar{u}$ controlled system to the set-point changes $0 \rightarrow 0.01 \rightarrow 1.0$.

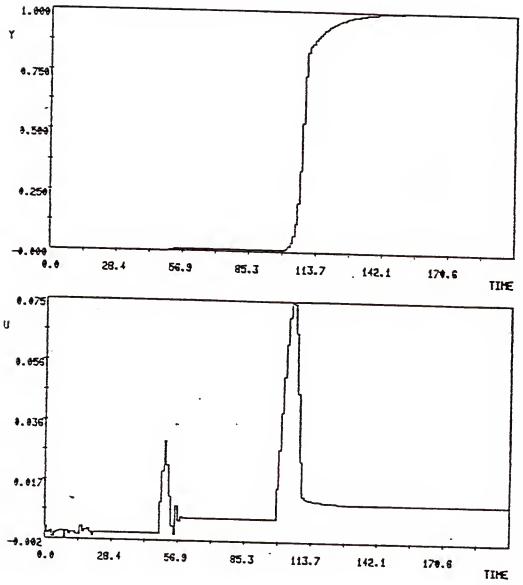


Figure 2-5. Response of the IMC controlled system to the set-point change 0.01 → 1.0.

CHAPTER III
A CAUTIOUS SELF-TUNING CONTROLLER

Introduction

This chapter presents a simple linear model based self-tuning controller, namely the Cautious Self-Tuning Controller (CSTC) which features enhanced robustness characteristics in the case of unmodeled nonlinearities and/or dynamics. The controller structure is that of a linear quadratic one minimizing a single-step cost function with output and input penalization. STCs involving input penalization have been proved to be more robust than the STR (Gawthrop and Lim, 1982) and are perhaps the most industrially accepted adaptive controllers (Dumont, 1986). The major drawback these controllers face is the requirement to know a priori weights that result in a closed-loop stable system. For open-loop stable systems stability and robustness can be achieved with heavy penalization of the control variable but this results in sluggish performance.

To circumvent the problem, algorithms involving time-varying weights tuned on-line have been presented. Allidina and Hughes (1980) calculate the weights on-line as the solution to pole placement related Bezoutian equations. Latawiec and Chyra (1983) utilize on-line stability criteria to adjust the weights. Toivonen (1983) interprets the input weight as the Langrange multiplier of a control variance constrained optimal control problem. These algorithms (all developed for SISO systems)

require a large amount of computations which may become prohibitive in the multivariable case.

The CSTC presented here is a simple self-tuning controller whose weight is tuned on-line based on the estimation error. The key idea is the incorporation of two parts in the control law, a minimum variance part which depends directly on the estimated parameters of an assumed linear model and a "caution" part which is independent of these estimated parameters. If the parameter estimates are poor, the "caution" part is dominant and drives the system slowly but safely towards the desired steady state. When the estimates improve to the point that the plant/model mismatch is negligible, the minimum variance part becomes dominant and rapidly brings the system at the target steady state without large oscillations.

The SISO version is presented in this chapter and the multivariable in Chapter IV.

The Cautious Self-Tuning Controller Structure

The Cautious Self-Tuning Controller is based on the same model as the STR given by equations (2-1)-(2-4) and (2-5)-(2-12). However, recognizing that the true process is probably nonlinear, CSTC is only partially dependent on this linear model, its influence disappearing whenever it fails to match the process behavior.

The CSTC is based on a performance index of the type

$$J(u(t)) = [\hat{y}(t+k+1|t) - y_s]^2 + q(t)\,[u(t) - u_c(t)]^2 \qquad (3\text{-}1)$$

where the "caution" term $u_c(t)$ is a conservative control law insensitive to or independent of the estimated model, and it is able to drive the system close to the desired set point. Possible choices of $u_c(t)$ are discussed in the next section. The future estimate $\hat{y}(t+k+1|t)$ is given by

$$\hat{y}(t+k+1|t) = x^T(t)\ \hat{\theta}(t) \tag{3-2}$$

where the estimate $\hat{\theta}(t)$ is obtained using a recursive estimator. A UDU factorization (Bierman, 1977) of a recursive least squares estimator with variable forgetting factor (Ydstie and Sargent, 1982; Fortescue et al., 1981) is well suited for identifying the linear part of a nonlinear process. The nonnegative weight $q(t)$ is not constant but an increasing function of the plant/model mismatch, becoming zero when the on-line identified model, describes the process perfectly.

Minimization of performance measure J with respect to u(t) results in

$$\hat{y}(t+k+1|t) - y_s + \frac{q(t)}{\hat{d}_0^2(t)}\,[u(t) - u_c(t)] = 0 \tag{3-3}$$

or, in view of (3-2) and (2-14), the CSTC control law is

$$u(t) = h(t)\ u_c(t) + [1-h(t)]\ u_{MV}(t) \tag{3-4}$$

where

$$h(t) = \frac{q(t)}{\hat{d}_0^2(t) + q(t)} \tag{3-5}$$

Since $0 \leq h(t) \leq 1$, the controller of (3-4) is a convex combination of the terms $u_c(t)$ and $u_{MV}(t)$. If the estimated parameter vector $\hat{\theta}(t)$ provides an accurate description of the system, then the weight $q(t)$ should be taken close to zero and $u_{MV}(t)$ would be the dominant term. In the case of large plant/model mismatch a large value of $q(t)$ is appropriate which would make the "caution" term $u_c(t)$ dominant and weigh out the erroneous $u_{MV}(t)$ contribution. A method for relating $q(t)$ to plant/model mismatch is given next.

Equation (3-3) may be written alternatively as

$$e(t+k+1) - \varepsilon(t+k+1|t) = -\frac{q(t)}{\hat{d}_0(t)}\,[u(t) - u_c(t)] \tag{3-6}$$

where

$$e(t) = y(t) - y_s \tag{3-7}$$

is the output error, and

$$\varepsilon(t|t-k-1) = y(t) - \hat{y}(t|t-k-1) \tag{3-8}$$

is the output prediction error given information up to time $t - k - 1$. If a local linear model is valid, the output error is

$$e(t+k+1) = y(t+k+1) - y_s = d_0[u(t) - u^*_{MV}(t)] \tag{3-9}$$

where $u^*_{MV}(t)$ is given by (2-13) (i.e. it is based on the true parameter values). Substitution of (3-9) into (3-6) gives

$$u(t) = h'(t)\, u_c(t) + [1-h'(t)]\, u^*_{MV}(t) + \frac{\hat{d}_0(t)h'(t)}{q(t)}\, \varepsilon(t+k+1|t) \qquad (3\text{-}10)$$

where

$$h'(t) = \frac{q(t)}{\hat{d}_0\hat{d}_0(t) + q(t)} \qquad (3\text{-}11)$$

The last term of (3-10) represents the erroneous part of the control law due to the inaccuracy of the on-line identified model. It is reasonable to limit that part to a fraction δ of the "caution" part, or

$$\left|\frac{\hat{d}_0(t)\, h'(t)}{q(t)}\, \varepsilon(t+k+1|t)\right| = \delta \left|h'(t)\, u_c(t)\right| \qquad (3\text{-}12)$$

from which follows

$$q(t) = \frac{|\hat{d}_0(t)|}{\delta|u_c(t)|}\, |\varepsilon(t+k+1|t)| \qquad (3\text{-}13)$$

The above equation expresses $q(t)$ as a linear function of the output prediction error $\varepsilon(t+k+1|t)$. Since this error is unknown at time t, it is substituted by a filtered $\varepsilon(t|t-k-1)$. The filtering, by not allowing large sudden drops in $q(t)$, protects against the case where, at some instant, parameter errors combine so as to produce a small $\varepsilon(t|t-k-1)$ even though the model is poor; in addition, by restricting the rate of change of $q(t)$, filtering smooths out the control law. Thus, the weight $q(t)$ is finally chosen as

$$q(t) = \frac{|\hat{d}_0(t)|}{\delta|u_c(t)|}\, v(t) \tag{3-14}$$

where

$$v(t) = pv(t-1) + (1-p)\,|\varepsilon(t|t-k-1)| \;;\quad 0 \le p < 1 \tag{3-15}$$

The corresponding h(t) is given by (3-5) as

$$h(t) = \frac{v(t)}{\delta|\hat{d}_0(t)\; u_c(t)| + v(t)} \tag{3-16}$$

One final note is in order. To assure that in (3-10) $h'(t) \in [0,1]$ we set $h(t) = 1$ (which implies $q(t) \to \infty$ and hence $h'(t) = 1$) whenever $\hat{d}_0(t)$ has the wrong sign. Thus, a priori knowledge of the sign of d_0 is required.

With the above choice of h(t) a large plant/model mismatch drives h(t) towards 1 and hence the CSTC control law (3-4) towards $u_c(t)$. As estimates improve the mismatch decreases, h(t) decreases and the control approaches $u_{MV}(t)$, as desired. However, it should be mentioned that the MV STC runs into stability problems if applied to systems with zeroes outside the open unit circle at the target steady state. Therefore, the CSTC should also not be used in such cases.

The "Caution" Term

In this section the CSTC design is completed by presenting two possible choices for the "caution" term $u_c(t)$.

The first and simplest possibility is

$$u_{c1}(t) = u_s + N(t) \tag{3-17}$$

where u_s is a constant approximating the control variable value at the target steady state. Such an approximate value is usually known from operating experience or from a steady-state model available from the design stage. Low level white noise excitation N(t) is added to aid the parameter estimator. Otherwise, identifiability problems which arise for a constant or constant parameter control law (Astrom and Wittenmark, 1973) would be encountered when h is high. The excitation component of u(t) will, of course, decrease as the estimated model improves and h drops.

If a constant weight q were used, an inaccurate u_s would result in steady-state offset. However, in the CSTC case q and hence the contribution of $u_{c1}(t)$ in u(t) decreases as the parameter estimates improve leading ultimately to zero or insignificant offset.

The second possible choise for $u_c(t)$ requires a priori knowledge of an approximate, perhaps very rough, dynamic linear model. The "caution" term is then designed as a very conservative IMC similarly to the "stabilizing" controller described in section 2.4. For example, if $B(q^{-1})$ has no zeroes outside the open unit circle, the internal model controller for (2-1) is

$$B(q^{-1})\,[1-fq^{-1} - (1-f)q^{-k-1}]\,u_{IMC}(t) = -(1-f)\,[1-q^{-1}A(q^{-1})]e(t) \tag{3-18}$$

If the filter pole f is chosen very close to 1, u_{IMC} is very conservative (and sluggish) but will drive the system to the desired steady state, even if the model is poor. This is exactly what is required for the "caution" term and therefore we set

$$u_{c2}(t) = u_{IMC}(t) + N(t) \tag{3-19}$$

It should be remarked here that a high value is recommended for f, even if the local model used to design $u_{IMC}(t)$ adequately represents the current state of process. Since plants are generally nonlinear, disturbance or set point changes may significantly alter the locally valid linear model. The sluggishness of $u_{IMC}(t)$, and hence $u_{c2}(t)$, is not a serious drawback for the CSTC as it might be for the "stabilizing" STC of the previous chapter since the $u_{MV}(t)$ component will speed up the system response as soon as the parameter estimates, aided by the excitation of $u_{c2}(t)$, improve.

The "caution" term $u_{c2}(t)$ is more difficult to obtain than $u_{c1}(t)$ but has the advantage that is guarantees complete elimination of steady-state offset even for h different than zero.

Example

The CSTC performance is demonstrated on the stirred tank reactor model given by Ahlgren and Stevens (1971). The reactor mass and energy balances are

$$\frac{dC}{dt} = \frac{Q}{V} C_i - \frac{Q}{V} C - Ke^{-E/RT}C \tag{3-20}$$

$$\frac{dT}{dt} = \frac{Q}{V} T_i - \frac{Q}{V} T + \frac{(-\Delta H)Ke^{-E/RT}}{\rho c_p} C - \frac{UA}{\rho c_p V} \frac{2\rho_c C_{pc} Q_c (T-T_c)}{2\rho_c c_{pc} Q_c + UA} \qquad (3\text{-}21)$$

The model parameter values are given in Ahlgren and Stevens (1971), and for notation consult Table 3-1.

The adaptive controllers tested here employ the UDU factorization of a recursive least squares with variable forgetting factors given in Appendix B and with parameters as in Table 3-2. To reduce numerical problems in the estimator the input and output variables are scaled so that the components of x(t-k-1) (Equation (2-13)) do not differ widely in order of magnitude. Thus,

$$y(t) = C(t)/C_{ref} \; ; \; u(t) = Q_c(t)/Q_{cref} \qquad (3\text{-}22)$$

where the reference values are given in Table 3-1. The table also includes all the CSTC parameters.

Figures 3-1 to 3-5 examine controller performance for small changes in the set point and the disturbance C_i. At time t = 450 (sampling intervals of 20s) the set point was changed from y_s = 0.05 to 0.10 and at t = 1200 the inlet concentration decreased by 10%. Figure 3-1 presents the performance of the MV STC. It is seen that it is fairly successful in following the set point and in rejecting the load change. However, the control variable behavior is unacceptably oscillatory. Figure 3-2 depicts the CSTC performance with "caution" term $u_{c1}(t)$. For both set points the a priori known u_s is initially 10% below the correct value and, since the inlet concentration drop is unmeasured, the u_s value is not altered at t = 1200 resulting in an increase of the u_s error to 58%. It is seen that both the output and input are considerably improved. The

last plot of Figure 3-2 is of h(t). After the set-point and disturbance changes, it increases reflecting the increase in plant/model mismatch, and subsequently drops as the parameter estimates improve. For comparison purposes Figure 3-3 shows the system response for $u(t) = u_s$. Since the u_s values are incorrect, offset results. The CSTC response for "caution" part $u_{c2}(t)$ is given in Figure 3-4. An approximate model is first obtained by estimating parameters while running the process around the initial steady state with white noise excitation. At t = 50 the CSTC starts and controls the reactor successfully. To be safe the filter constant was assigned the high value f = 0.999. For this value the IMC response is extremely sluggish as seen in Figure 3-5.

Figures 3-6 to 3-10 show controller performance for the large set-point change from 0.95 to 0.10. The MV STC, as seen in Figure 3-6, completely fails to drive the process to the new set point. Instead, the coolant flowrate is stuck at the lower bound. In contrast, the SCTC with "caution" $u_{c1}(t)$ succeeds (Figure 3-7), even though the a priori specified u_s if off by 10%. It is worth noting that the reactor has steady-state multiplicity and for control $u(t) = u_{c1}(t)$ the process winds up at the wrong steady state (Figure 3-8). The successful performance of the CSTC with $u_{c2}(t)$ is shown in Figure 3-9. Figure 3-10 depicts the sluggish response of $u(t) = u_{c2}(t)$. Finally, it should be mentioned that the CSTC responds quite well to step changes in any of the disturbances. For example, Figure 3-11 shows the CSTC response to the changes (i) 20% drop in C_i at t = 300; (ii) 50% increase in C_i at t = 600; (iii) 5% drop in T_i (°K) at t = 900; (iv) 8% drop in T_c at t = 1200; (v) 11% increase in Q at t = 1500, and (vi) 9% drop in UA at t = 1800.

Remark. It should be pointed out that the CSTC, by slowing down the system and adding excitation, aids the parameter estimator. As a result the model identified by the CSTC differs from that identified by the MV STC even though both have been tuned identically, and when h approaches zero the CSTC control law performs considerably better than the MV STC law.

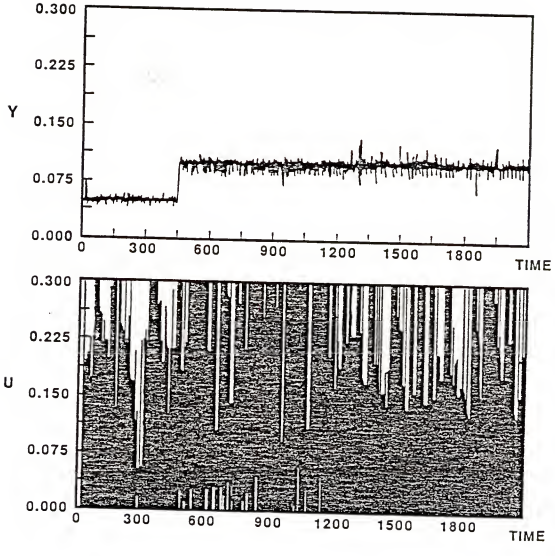


Figure 3-1. Dimensionless concentration y(t) and coolant flowrate u(t) when the reactor is controlled by the MV STC.

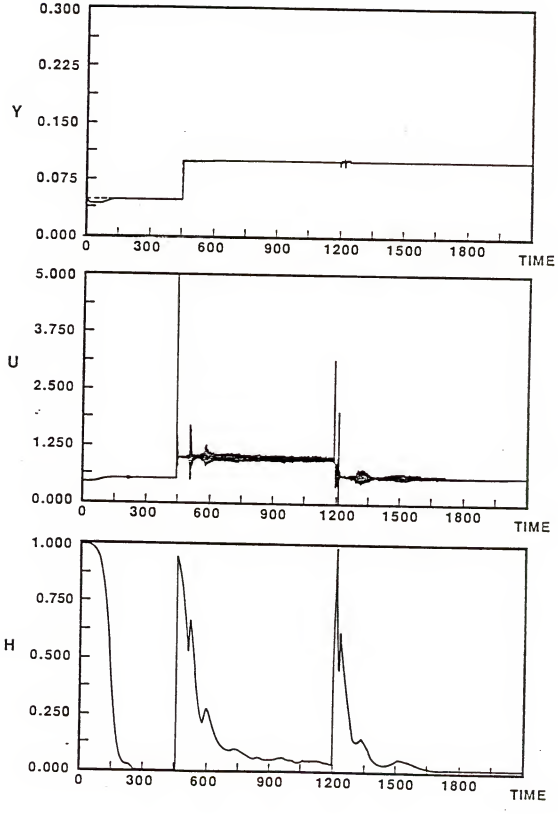


Figure 3-2. Dimensionless concentration y(t), coolant flowrate u(t) and weight h(t) when the reactor is controlled by the CSTC with caution term $u_{cl}(t)$.

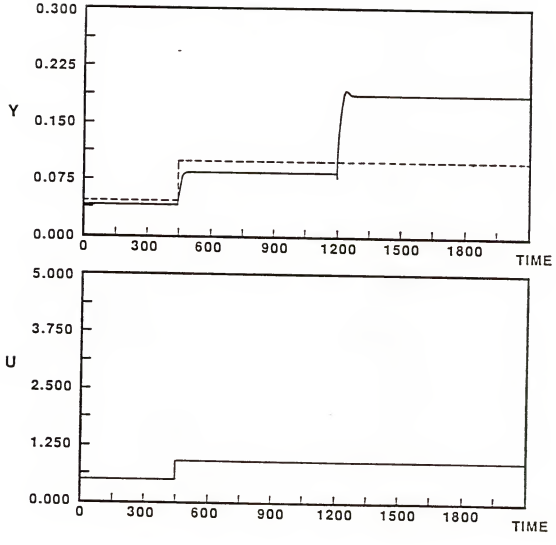


Figure 3-3. Dimensionless concentration y(t) and coolant flowrate u(t) when the reactor is run open loop with $u(t) = u_s$. The dotted line is the set-point.

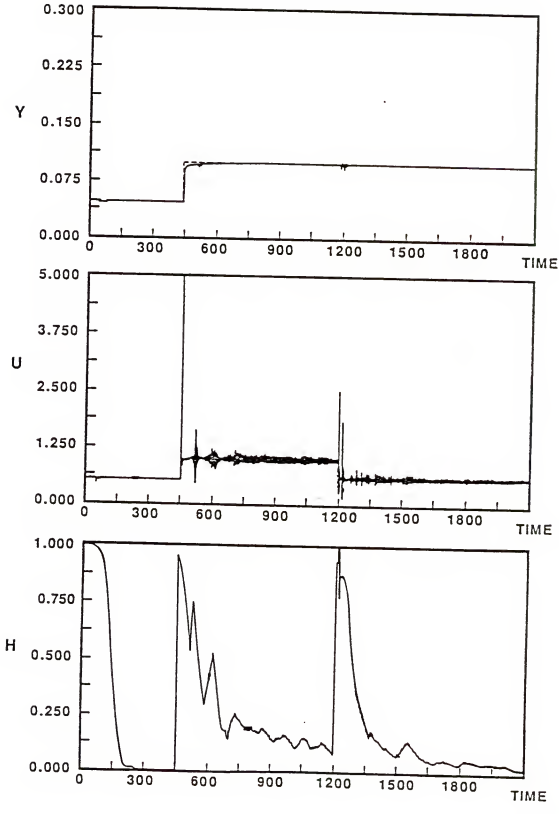


Figure 3-4. Dimensionless concentration y(t), coolant flowrate u(t) and weight h(t) when the reactor is controlled by the CSTC with caution term $u_{c2}(t)$.

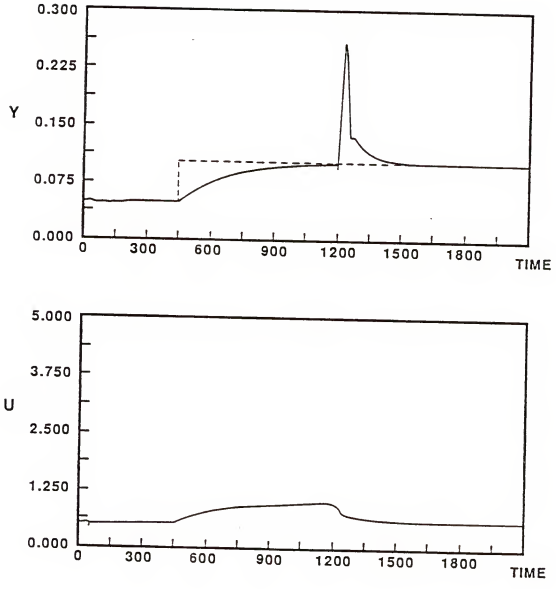


Figure 3-5. Dimensionless concentration y(t) and coolant flowrate u(t) when the reactor is controlled by $u_{IMC}(t)$.

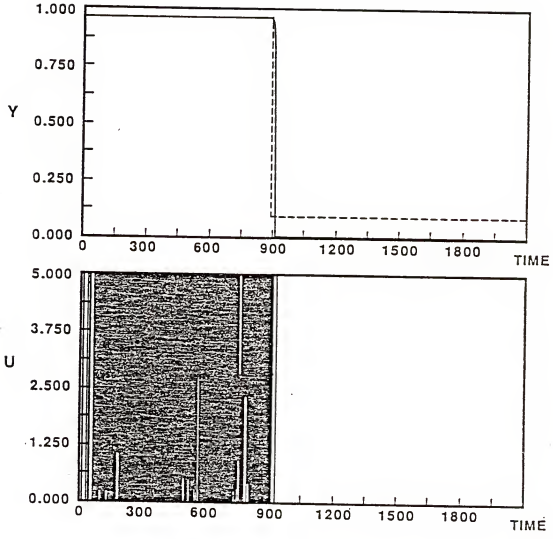


Figure 3-6. Dimensionless concentration y(t) and coolant flowrate u(t) when the reactor is controlled by the MV STC.

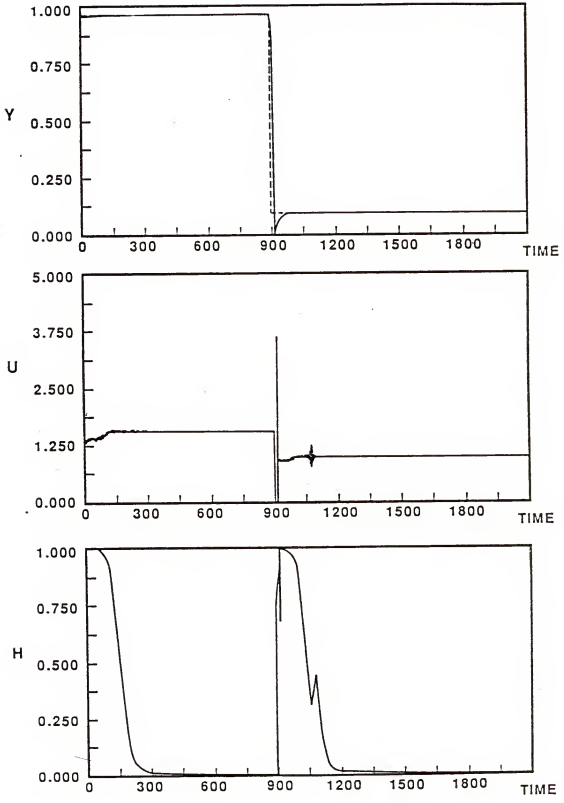


Figure 3-7. Dimensionless concentration y(t), coolant flowrate u(t) and weight h(t) when the reactor is controlled by the CSTC with caution term $u_{cl}(t)$.

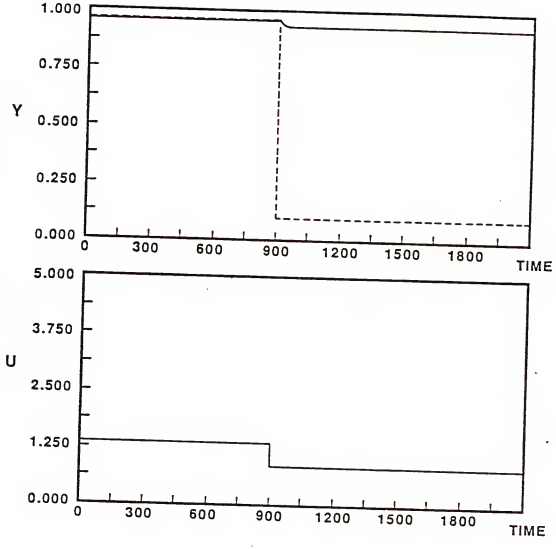


Figure 3-8. Dimensionless concentration y(t) and coolant flowrate u(t) when the reactor is run open loop with u(t) = u_s.

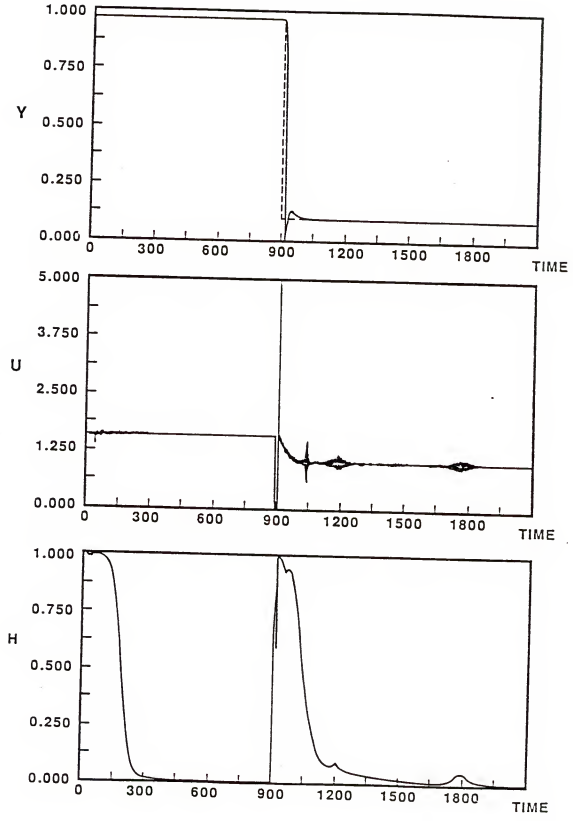


Figure 3-9. Dimensionless concentration y(t), coolant flowrate u(t) and weight h(t) when the reactor is controlled by the CSTC with caution term $u_{c2}(t)$.

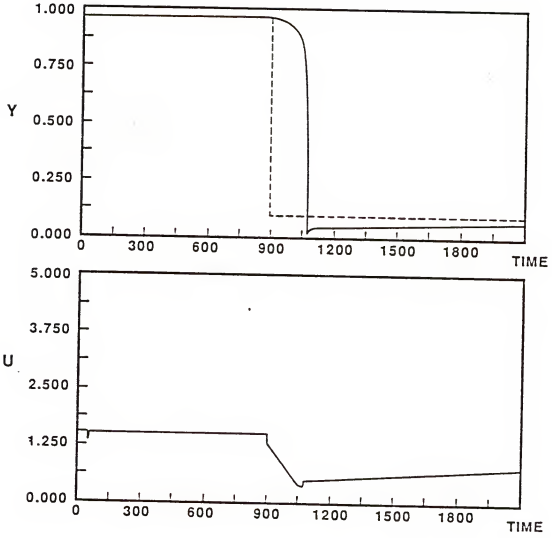


Figure 3-10. Dimensionless concentration y(t), coolant flowrate u(t) when the reactor is controlled by $u_{IMC}(t)$.

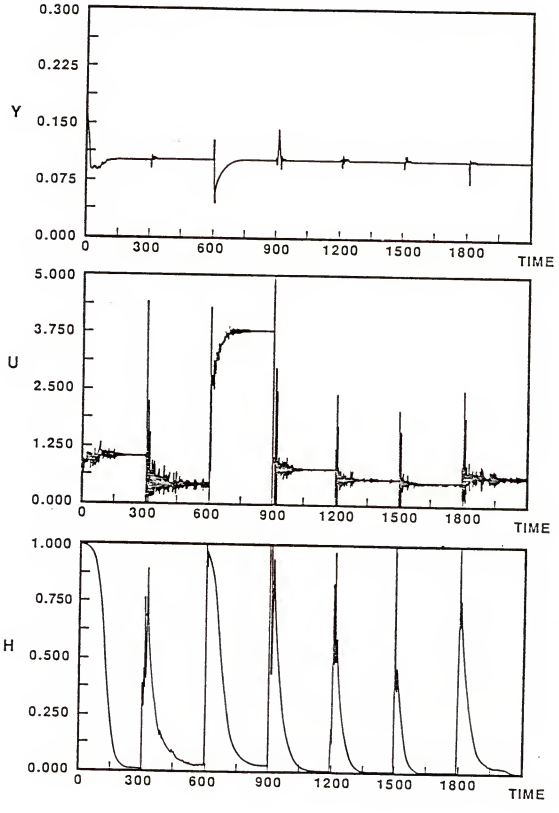


Figure 3-11. Dimensionless concentration y(t), coolant flowrate u(t) and weight h(t) when the reactor is controlled by the CSTC with caution term $u_{cl}(t)$.

Table 3-1

Notation for Example

A	= heat exchange area
C	= concentration
C_i	= inlet concentration
c_p	= reactant specific heat capacity
c_{pc}	= coolant specific heat capacity
C_{ref}	= reference concentration
E	= activation energy
ΔH	= enthalpy change of reaction
K	= reaction rate preexponential factor
Q	= inlet flowrate
Q_c	= coolant flowrate
Q_{cref}	= reference coolant flowrate
R	= gas constant
T	= reactor temperature
T_c	= coolant temperature
T_i	= inlet temperature
U	= heat transfer coefficient
V	= reactor volume

Greek Letters

ρ	= reactant density
ρ_c	= coolant density

Table 3-2

Parameter Values

Estimator Parameters

$\gamma = 1 \times 10^{-5}$

$\sigma = 1 \times 10^{-5}$

$\lambda_{min} = 0.1$

Scaling Parameters

$C_{ref} = 0.5$ lb-moles/ft^3 = 8.01 kg-moles/m^3

$Q_{cref} = 0.2$ ft^3/s = 5.664×10^{-3} m^3/s

CSTC Parameters

$\delta = 0.20$

$p = 0.95$

$E\{\xi^2(t)\} = 0.01$

u_s (when $y_s = 0.05$) = 0.05 (correct value = 0.573)

u_s (when $y_s = 0.10$) = 0.9 (correct value = 1.02)

u_s (when $y_s = 0.95$) = 1.4 (correct value = 1.57)

$f = 0.999$

CHAPTER IV
A MIMO CAUTIOUS SELF-TUNING CONTROLLER

Introduction

In this chapter the Cautious Self-Tuning Controller is extended for multi input-multi output (MIMO) systems with multiple time delays. MIMO quadratic STCs have been developed first by Borisson (1979), Keviczky and Kumar (1982), and Koivo (1980) assuming the same time delay for each input-output pair. Subsequently Tsiligiannis and Svoronos (in press; 1986b) presented quadratic STCs applicable to systems with multiple delays with non-trivial cross-couplings. All these controllers are based on constant weights penalizing the control variable with the associated problems discussed in section 3.1

Here the control weights are tuned on-line so as to maintain the input and output variables bounded despite any unmodeled dynamics, nonlinearities and/or estimation error.

The design and implementation of the controller maintains the essential simplicity and small computational requirements of the STR.

Model

A discrete time input-output representation of linear multivariable systems has the form

$$A'(q^{-1})Y(t+1) = B'(q^{-1})U(t) + S' \qquad (4-1)$$

where Y(t), U(t) are the l-dimensional input and output vectors, q^{-1} is the backward shift operator, and $A'(q^{-1})$, $B'(q^{-1})$ are polynomial matrices in q^{-1}. The l-dimensional vector S' represents deterministic disturbances which are either slowly varying or experiencing step changes and can be identified by a suitable estimator.

In particular it can be assumed without loss of generality that

$$A'(q^{-1}) = \text{diag}\,[a_i(q^{-1})]$$

with

$$a_i(q^{-1}) = 1 + a_{i1}\, q^{-1} + \ldots + a_{i\alpha_i}\, a^{-\alpha_i}$$

and

$$(B'(q^{-1}))_{ij} = q^{-d_{ij}}\, b'_{ij}(1^{-1})$$

with

$$b'_{ij}(q^{-1}) = b'_{ij0} + b'_{ij1}\, q^{-1} + \ldots + b'_{ij\beta_{ij}}\, q^{-\beta_{ij}}$$

$$b'_{ij0} \neq 0 \qquad \forall_{i,j}$$

and d_{ij} is the time-delay between the i^{th} output and the j^{th} input.

In designing one-step-ahead controllers the predictor form of (4-1) is needed. This step is not trivial in the presence of multiple delays and has not been resolved yet for the general case. However, there exist methods (requiring apriori knowledge for the delay structure of $B'(q^{-1})$) which recast (4-1) into

$$A(q^{-1})\, W(t+d+1) = B(q^{-1})\, V(t) + S \qquad (4\text{-}2)$$

with

$$A(q^{-1}) = \sum_{i=1}^{n+1} A_i\, q^{-i}\ , \quad A_0 = I \tag{4-3}$$

$$B(q^{-1}) = \sum_{i=1}^{m} B_i\, q^{-i}\ , \quad \det(B_0) \neq 0 \tag{4-4}$$

$$W(t) = q^{-k}\, K(q)\, Y(t) \tag{4-5}$$

$$V(t) = q^{\mu}\, M(q^{-1})\, U(t) \tag{4-6}$$

and

$$d = k + \mu$$

where $K(q)$, $M(q^{-1})$ are polynomial matrices in q and q^{-1}, respectively, and k, μ are the maximum degrees in any element of $K(q)$, $M(q^{-1})$, respectively. The vector $W(t)$ is known at time t while $U(t)$ is determined from the present and previous $V(t)$. The matrices $K(q)$, $M(q^{-1})$ are known and depend on the available information. It is required that $[q^{\mu}\, M(q^{-1})]^{-1}$ be a stable operator.

For example, if the left interactor matrix $L(q)$ of model (4-1) is known (Goodwin and Sin, 1984, p. 132), then

$$M(q^{-1}) = I\ , \quad \mu = 0$$

$$K(q) = L(q)$$

If the right interactor matrix R(q) is known (Tsiligiannis and Svoronos, 1986b), then

$$K(q) = I \qquad k = 0$$

$$M(q^{-1}) = R(q^{-1})$$

Both of the above methods require the knowledge of the interactors L(q) or R(q) which can be restrictive in many cases. A method requiring the knowledge of only the time delays d_{ij} has been presented by Tsiligiannis and Svoronos (in press). According to this method

$$K(q) = \text{diag}\,(q^{k_i}) \qquad k = \max\,(k_i)$$

$$M(q) = \text{diag}\,(q^{\mu_i}) \qquad \mu = \max\,(\mu_i)$$

and the k_i, μ_i are found from the d_{ij}'s by solving an integer programming problem. The method can treat large classes of systems with nontrivial cross couplings, but not the general case. An alternative method for a similar class of systems was proposed by Singh and Narendra (1984).

The aim of all these methods is to obtain a parameterization model (4-2) of (4-1) with $\det(B_0) \neq 0$ and $[q^{\mu} M(q^{-1})]^{-1}$ stable. Then the predictor form is obtained from (4-2) using the identity

$$I = F(q^{-1})\, A(q^{-1}) + q^{-d-1}\, G(q^{-1}) \qquad (4\text{-}7)$$

where

$$F(q^{-1}) = I + F_1 q^{-1} + \ldots + F_d q^{-d} \tag{4-8}$$

$$G(q^{-1}) = G_0 + G_1 q^{-1} + \ldots + G_n q^{-n} \tag{4-9}$$

and

$$W(t+d+1) = C(q^{-1}) W(t) + D(q^{-1}) V(t) + r \tag{4-10}$$

where

$$C(q^{-1}) = G(q^{-1})$$

$$D(q^{-1}) = F(q^{-1}) B(q^{-1})$$

and

$$D_0 = B_0$$

Equation (4-10) is equivalently written in vector form as

$$W(t+d+1) = \theta X(t)$$

$$= \theta_1 X_1(t) + B_0 V(t) \tag{4-11}$$

where θ is the model parameter matrix,

$$X^T(t) = [W^T(t)\ W^T(t-1) \ldots W^T(t-n)\ \ V^T(t)\ V^T(t-1) \ldots V^T(t-m-d)1]$$

and

$$X_1^T(t) = [W^T(t)\ W^T(t-1)\ \ldots\ W^T(t-n)\ \ V^T(t-1)\ \ldots\ V^T(t-m-d)1]$$

The unknown model parameters θ are estimated on-line from (4-11) via a recursive estimator. In particular, we consider here a multivariable version of the least-squares estimator presented in Appendix B.

Define the parameter error matrix

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t)$$

Then, the output prediction at time $t + d + 1$ given information up to time t is

$$\hat{W}(t+d+1|t) = \hat{\theta}(t)\ X(t) \tag{4-12}$$

$$= \hat{\theta}_1(t)\ X_1(t) + \hat{B}_0(t)\ V(t)$$

Also, the output prediction at time t given parameter estimates at time $t - 1$ is

$$\hat{W}(t|t-1) = \hat{\theta}(t-1)\ X(t-d-1) \tag{4-13}$$

and the output prediction error

$$\varepsilon(t) = W(t) - \hat{W}(t|t-1) \tag{4-14}$$

The CSTC requires that $\hat{B}_0(t)$ is non-singular. The time instants at which $\hat{B}_0(t)$ becomes singular form a set of measure zero. But even for those instants, it is shown in Goodwin and Sin (1984, p. 206) that a simple algorithm modification removes the problem.

Multivariable Cautious Self-Tuning Controller

The Cautious Self-Tuning Controller (CSTC) is based on the following single stage cost function:

$$J = |\hat{W}(t+d+1|t) - W_r|^2 + [V(t) - V_r]^T Q(t) [V(t) - V_r] \qquad (4\text{-}15)$$

where $Q(t)$ is a positive semidefinite matrix, W_r is a filtered reference signal given by

$$W_r = K(q) Y_r$$

Also

$$V_r = M(1) U_r$$

where U_r is the steady-state control vector corresponding to Y_r (or an approximation to it). Equation (4-15) differs from the cost function introduced in Clarke and Gawthorp (1975), in that $Q(t)$ is allowed to be time-varying and is tuned on-line.

Upon minimization with respect to $V(t)$, (4-15) results in the following expression for the control law

$$[\hat{\theta}_1(t)X_1(t)-W_r] + [\hat{B}_0(t)+\hat{B}_0^{-T}(t)Q(t)]V(t) = \hat{B}_0^{-T}(t)Q(t)V_r \qquad (4\text{-}16)$$

where the superscript -T denotes transposed inverse. If $Q(t) = 0$ at time t, the one-step-ahead dead-beat control policy is obtained

$$V_{db}(t) = \hat{B}_0^{-1}(t) \, [W_r - \hat{\theta}_1(t) \, X_1(t)] \tag{4-17}$$

If $Q(t) = 0 \; \forall \; t$, then (4-17) corresponds to the optimal dead-beat control law (minimum variance in the stochastic case).

Using (4-16) and (4-17) we can express the cautious control law V(t) as a linear combination of the constant term V_r and the dead-beat control V_{db}

$$V(t) = H(t) \, V_r + [I - H(t)] \, V_{db}(t) \tag{4-18}$$

where

$$H(T) = [\hat{B}_0^T(t) \, \hat{B}_0(t) + Q(t)]^{-1} \, Q(t) \tag{4-19}$$

The V_{db} term represents a "fast" performing control law which if based on the true process model achieves $W(t+d+1) = W_r$. But if it is based on a poor model it might lead to poor performance. On the other hand, the term V_r is model independent and drives the system outputs close to Y_r with open-loop dynamics. It represents a "worst case" robust control law. The positive semi-definite matrix H(t) weighs the contributions of the "fast" V_{db} and the robust V_r terms in (4-18).

In what follows we choose H(t) to be a diagonal matrix; i.e.

$$H(t) \equiv \text{diag} \, [h_1(t), \, h_2(t), \, \ldots, \, h_1(t)] \tag{4-20}$$

with

$$0 \leq h_i(t) < 1 \qquad i = 0, \ldots, l$$

Note that since (4-19) gives

$$Q(t) = [I-H(t)]^{-1} \hat{B}_0^T(t) \hat{B}_0(t) H(t)$$

Q(t) is positive semidefinite for the above choise of H(t).

The structural simplicity of (4-18) leads to an easier and more transparent way of tuning the algorithm through H(t) instead of Q(t). In the case of large plant/model mismatch a large value of $h_i(t)$ close to one is appropriate. This choice would make the robust term dominant and weigh out the unreliable V_{db} contribution. When the estimated parameter matrix $\hat{\theta}(t)$ and (4-12) provide an accurate description of the process, $h_i(t)$ should be taken close to zero making V_{db} the dominant term.

A method for relating $h_i(t)$ to the plant/model mismatch is given next. The objective is to obtain a control law which is bounded in spite of modeling inaccuracy and estimation errors.

From (4-18) we obtain

$$V(t) - V_r = (I - H(t)) (V_{db}(t) - V_r) \tag{4-21}$$

Define

$$\tilde{V}(t) = V(t) - V_r = [\tilde{V}_1(t), \ldots, \tilde{V}_1(t)]^T$$

and

$$\tilde{V}_{db}(t) = V_{db}(t) - V_r = [\tilde{V}_{db1}(t), \ldots, \tilde{V}_{db1}(t)]^T$$

Then from (4-21)

$$\tilde{V}_i(t) = (1 - h_i(t))\, \tilde{V}_{dbi}(t) \qquad i = 1, \ldots, l$$

or

$$|\tilde{V}_i(t)| = (1 - h_i(t)\, |\tilde{V}_{dbi}(t)| \qquad i = 1, .., l \tag{4-22}$$

The following bound is imposed on $|\tilde{V}_i(t)|$

$$|\tilde{V}_i(t) \leq \frac{\delta_i}{\eta(t)} \qquad i = 1, \ldots, l \tag{4-23}$$

where $\eta(t)$ is a measure of the plant/model mismatch.

A large $\eta(t)$ restricts the control $V_i(t)$ close to V_{ri} and effectively weighs out erroneous contributions from $V_{dbi}(t)$. As the model improves, $\eta(t)$ decreases allowing larger control amplitudes and the control law achieves faster performance.

Equation (4-23) is satisfied if

$$h_i(t) = \frac{\eta(t)}{\dfrac{\delta_i}{|\tilde{V}_{dbi}(t)|} + \eta(t)} \tag{4-24}$$

which is functionally similar to (3-16) and gives $h_i(t) = 0$ only for $\eta = 0$.

The same measure of plant/model mismatch as the one introduced in Chapter III (3-15) is used here. It is related to the prediction error by

$$\eta(t) = \eta(t-1) + (1-\lambda)\,|\varepsilon(t)| \;;\quad 0 < \lambda < 1 \qquad (4\text{-}25)$$

Note that the method presented in Chapter III for tuning h(t) can be easily extended to multivariable systems. This method is based on the minimization of control law sensitivity to the prediction error. In this chapter H(t) tuning is based on a different method. The objective is to keep the control bounded in case of large errors. Both methods express $h_i(t)$ as increasing function of the plant/model mismatch. It should be pointed out here that other objectives may be used in tuning H(t) as well as different measures of the plant/model mismatch, e.g. the aposteriori prediction error or the estimator quantity $|\varepsilon(t+1)|^2 / (\lambda(t+1) + w(t))$.

Stability and Robustness of CSTC

The CSTC has been designed with non-ideal applications in mind. It will be shown that under assumptions satisfied by many nonlinear systems the input and output variables remain bounded.

Statement 4.1

Assume

1. that $|\varepsilon(t)|$ cannot converge to zero unless the output is bounded or unless the input is bounded,
2. that the true process model is bounded input-bounded output stable, i.e.

$$||y(t)||_\infty \leq K_1 + K_2 \, ||u(t)||_\infty \tag{4-26}$$

and bounded output bounded input stable, i.e.

$$||u(t)||_\infty \leq K_1 + K_2 \, ||y(t)||_\infty \tag{4-27}$$

where $||\cdot(t)||_\infty$ denotes the l_∞ norm truncated at time t, i.e.

$$||\cdot(t)||_\infty = \max_{0 \leq \tau \leq t} |\cdot(\tau)| \quad ;$$

then, if the process is controlled by the CSTC the input and output variables remain bounded.

Proof. Assume that the output is unbounded. Since $|\varepsilon(t)|$ cannot converge to zero (assumption 1) it follows from (4-25) that $\eta(t)$ is bounded away from zero. Inequality (4-23) then implies that v(t) is bounded and since $q^{\mu} M(q^{-1})$ is stable it follows from (4-6) that u(t) is bounded. Then it follows from (4-26) that y(t) is bounded thus contradicting the initial assumption. Therefore, y(t) is bounded and from (4-27) u(t) is also bounded.

The same result is obtained in analogous manner if we start with the assumption that the input is bounded.

Remark. For linear systems (4-26) means that the system is open-loop stable and (4-27) that is minimum phase.

CHAPTER V
MODEL DEPENDENT CONTROL ANALYSIS

Introduction

Controller design methods for linear systems can be distinguished in two general categories. Fixed structure methods where the controller structure is first selected and then any free parameters are tuned based on process information. Advantages of the method are its simplicity and that it does not require knowledge of a detailed model. The major disadvantage is poor closed-loop performance in many cases. The best known controller in this category is the PID with three tuning parameters. The second category consists of methods which require the knowledge of a detailed process model in order to design the controller. Here one first specifies the closed-loop performance, and then the controller structure and parameters follow from the design method and the process model. This category includes optimal control, state feedback, pole/zero placement methods and numerous others.

An interesting method of designing model dependent controllers has been developed recently and is based on model inversion. The general approach includes Internal Model Control (Garcia and Morari, 1982, 1985a, 1985b; Morari, 1983; Morari et al., 1984; Economou et al., 1984), Inferential Control (Brosilow and Tong, 1978; Brosilow, 1979; Chen and Brosilow, 1984), Q-parametrization (Zames, 1981; Desoer and Chen, 1981). All these methods are based on the same design philosophy with

differences on the final control realization. The design procedure consists of obtaining an approximate stable model inverse and then taking the controller as the product of this inverse and a filter which is designed to achieve the desired performance and robustness. This simple and elegant procedure gives excellent results with respect to set-point tracking but might not perform well in rejecting disturbances especially if the open-loop dynamics are slow. Also, the method is applicable only to open-loop stable systems.

The Model Dependent Controller (MDC) introduced recently by Svoronos (1986) handles effectively both set-point tracking and disturbance rejection, and is applicable to both open-loop stable and unstable systems. The method first transforms the input so that the system is stable with respect to the transformed input and with disturbance poles at more favorable locations. Then the IMC design methodology is used for set-point tracking. The algorithm resolves the robustness versus performance issue in a simple and straightforward way through two tuning parameters which determine the location of the closed-loop poles.

Some modifications of the MDC algorithm are presented in this chapter which improve the closed-loop performance. Also, the MDC robustness is analyzed and it is proved that the algorithm is robust under model uncertainty. Only SISO systems are treated in this chapter.

The MDC Method

Consider the discrete time linear model

$$A(z)\,\bar{y}(z) = B(z)\,\bar{u}(z) + C(z)\,\bar{d}(z) \tag{5-1}$$

where $\bar{y}(z)$, $\bar{u}(z)$, $\bar{d}(z)$ are the z-transforms of the output $y(t)$, the control $u(t)$ and the disturbance $d(t)$, respectively, and $A(z)$, $B(z)$, $C(z)$ are polynomials in the complex variable z^{-1}. Model (5-1) can be expressed equivalently in transfer function form as

$$\bar{y}(z) = G(z)\,\bar{u}(z) + G_d(z)\,\bar{d}(z) \tag{5-2}$$

where

$$G(z) = \frac{B(z)}{A(z)}$$

$$G_d(z) = \frac{C(z)}{A(z)}$$

As mentioned earlier, inversion based methods, e.g. IMC, require that $G(z)$ is stable. No such assumption is made here. Instead the system is first "stabilized" by feedback and then the model inversion methodology is applied.

This is achieved by defining a generalized input v as function of the control u and the output y; this will be selected so that the poles of the transfer function between y and v are in desired locations. Subsequently the generalized input will be designed using the IMC methodology.

Stabilization

The polynomial B(z) is factored as

$$B(z) = B_+(z)\, B_-(z)$$

where $B_+(z)$ contains the non-invertible elements of B(z), i.e. the time delay and all zeroes of B(z) on or outside the unit circle. The generalized input is then defined from

$$\bar{v}(z) = Q(z)\, B_-(z)\, \bar{u}(z) + P(z)\, \bar{y}(z) \qquad (5\text{-}3)$$

where Q(z), P(z) are polynomials that will be specified later. In the following, the argument z is dropped for brevity. It may appear however when needed to enhance clarity. Substitution of (5-3) in (5-1) results to

$$T\bar{y} = B_+\bar{v} + QC\bar{d} \qquad (5\text{-}4)$$

where

$$T = AQ + B_+P \qquad (5\text{-}5)$$

The polynomial T is a design quantity whose zeroes are the poles of the transfer function between $\bar{y}$ and $\bar{v}$.

The polynomials Q and P are found as the solution of the Bezoutian equation (5-5). This is solvable provided that B_+ and A are coprime (or the system is stabilizable).

Given P and Q, (5-3) represents a linear discrete control law where ∇ plays the role of a reference signal. Next, ∇ will be formulated as a function of the known reference signal, i.e. the output set-point y_{sp}.

Generalized Input Design via IMC

If we consider (5-4) as the process model, the IMC law is given by the methods described in Appendix A as

$$\nabla = \frac{TF}{(1-B_+F)} [y_{sp} - y] \tag{5-6}$$

where F is the filter transfer function which without loss of generality can be expressed as

$$F(z) = \frac{F_N(z)}{F_D(z)} \tag{5-7}$$

The polynomials $F_N(z)$, $F_D(z)$ will be specified in the following section.

MDC Control Law

The MDC control law is obtained combining (5-6) and (5-3) as

$$H\,u + Ky = F_N\,T\,y_{sp} \tag{5-8}$$

where

$$H = QB_- (F_D - B_+F_N)$$

$$K = QAF_N + PF_D$$

Closed-Loop Equations

Using (5-8) and (5-1) we obtain the closed-loop equations

$$T_c \begin{bmatrix} \bar{y} \\ \bar{u} \end{bmatrix} = F_N T \begin{bmatrix} B \\ A \end{bmatrix} \bar{y}_{sp} + C \begin{bmatrix} H \\ -K \end{bmatrix} \bar{d}$$

where

$$T_c = AH + BK = B_-F_DT \tag{5-10}$$

It is seen from (5-9) and (5-10) that the closed-loop system is stable. Cancelling known and stable poles and zeroes in (5-9) we obtain

$$\bar{y} = \frac{B_+F_N}{F_D} \bar{y}_{sp} + \frac{(F_D - B_+F_N)QC}{TF_D} \bar{d} \tag{5-11}$$

It is seen that the closed-loop disturbance dynamics include the IMC filter poles. If it is desired that the tracking and disturbance rejection dynamics be independent, F_D is cancelled by taking

$$Q = Q_1 F_D \tag{5-12}$$

and (5-11) becomes

$$\bar{y} = \frac{B_+F_N}{F_D}\bar{y}_{sp} + \frac{(F_D - B_+F_N)Q_1C}{T}\bar{d} \tag{5-13}$$

It is clear from (5-13) that the closed-loop performance for set-point tracking depends on F_D and that for disturbance rejection only on T. Both F_D and T are designed polynomials. As in IMC, for asymptotically constant disburbances MDC eliminates steady-state offset if

$$F_D(1) - B_+(1)\ F_N(1) = 0 \tag{5-14}$$

or

$$\frac{B_+(1)\ F_N(1)}{F_D(1)} = 1 \tag{5-15}$$

as it can be easily verified from (5-13).

Closed-Loop Polynomial Selection

The transient performance of the closed-loop system depends on the location of the closed-loop poles. As shown later in section 5.5 the MDC robustness also depends on the pole location. In this section, the closed-loop pole selection is discussed.

Set-Point Tracking Poles

Robustness and performance analysis for IMC (Appendix A) results in the following selection for the tracking poles:

$$F_D(z) = \bar{B}_+(z)\,(1-fz^{-1}) \qquad 0 \le f < 1 \tag{5-16}$$

with

$$\bar{B}_+(z) = \prod_{i=1}^{m} (-z_i^+ + z^{-1}) \tag{5-17}$$

where z_i^+ ($i = 1,\ldots,m$) are the zeroes of $B_+(z)$. Note that $B_+(z)$ and $\bar{B}(z)$ form an "allpass"; i.e.

$$\left|\frac{B_+(z)}{\bar{B}_+(z)}\right| = 1 \quad \text{for} \quad z = e^{j\omega}$$

and the zeroes $\dfrac{1}{z_i^+}$ ($i = 1, \ldots, m$) of $B_+(z)$ are all inside the unit circle.

In view of (5-15), the steady-state offset is eliminated if

$$F_N = 1-f \tag{5-18}$$

Using (5-16), (5-17), and (5-18) the tracking properties are described by

$$\bar{y}(z) = \frac{B_+(z)}{\bar{B}_+(z)} \; \frac{1-f}{1-fz^{-1}} \; \bar{y}_{sp}(z)$$

which gives

$$|\bar{y}(z)| = \frac{1-f}{|1-fz^{-1}|}\ |\bar{y}_{sp}(z)| \quad \text{at} \quad |z| = 1$$

Disturbance Rejection Poles

Disturbance rejection depends on the zeroes of T(z) which in view of (5-5) and (5-16) becomes

$$T = A_1 Q_1 + B_+ P \tag{5-19}$$

where

$$A_1(z) = A(z)\ F_D = a_0 \prod_{i=1}^{n_1} (1-p_i z^{-1})$$

and p_i ($i = 1, \ldots, n_1$) are the zeroes of $A_1(z)$. These zeroes include the model open-loop poles, the zeroes $1/z_j^+$ ($j, 1, \ldots, m$) of $\bar{B}_+(z)$ and the filter pole of f.

Then, T(z) is chosen as

$$T(z;r) = a_0 \prod_{i=1}^{n_1} \left(1-r\frac{p_i}{p_{max}} z^{-1}\right) \quad 0 \le r \le 1 \tag{5-20}$$

where

$$p_{max} = \max \left\{1, \left(\max_i |p_i| + \varepsilon\right)\right\} \qquad i = 1, \ldots, n_1$$

for some small $\varepsilon \geq 0$, and with $\varepsilon = 0$ for open-loop stable systems. Alternatively $T(z;r)$ can be obtained without having to solve for the open-loop poles from

$$T(z;r) = A_1 \left(\frac{P_{max}}{} z\right) \tag{5-21}$$

We see from (5-20) that the closed-loop poles lie inside concentric circles of radii

$$R = r\, R_{max} \qquad 0 \leq r \leq 1$$

where $R = R_{max}$ is a circle containing all the open-loop model and filter poles scaled by P_{max}. The scaling ensures that the zeroes of $T(z)$ are inside the unit circle when the system is open-loop unstable. The concentric circles of radius R represent loci of constant settling times. It is easy to show that for $r = 0$ MDC is equivalent to the dead-beat controller and for $r = 1$ to IMC (for open-loop stable systems).

MDC Algorithm

Collecting all the pieces together, the MDC algorithm is summarized as follows:

(i) Choose parameters f and r so the desired performance and robustness specifications are met.

(ii) Choose the IMC filter as

$$F(z) = \frac{F_N}{F_D(z)}$$

$F_D(z) = \bar{B}_+(z)\ (1-fz^{-1})$

$F_N = 1-f$

(iii) Find T(z) from

$$T(r,z) = A_1 \left(\frac{rz}{P_{max}}\right)$$

where

$A_1(z) = A(z)\ F_D(z)$

and solve for Q_1 and P from

$T(z) = A_1(z)\ Q_1(z) + B_+(z)\ P$

This equation has an infinite number of solutions and the minimal order one is obtained by setting the degrees of P(z) and Q(z) to

$deg(P) = deg(A_1)-1$

$deg(Q_1) = deg(B_+)-1$

(iv) Obtain the MDC law from

$$F_D Q_1 B_-(F_D - B_+F_N)\bar{u} + F_D(Q_1 A F_N + P)\ \bar{y} = F_N T y_{sp}$$

It would be desirable in many cases that MDC involves only one tuning parameter. This can be achieved by taking

$f = r$

and

$Q = \bar{B}_+\ Q_1$

i.e. the filter pole z = f is not cancelled in the second term of the RHS of (5-11).

This makes MDC a very practical algorithm with only one parameter to tune.

Robustness of MDC Law

In many cases the available model is only an approximate description of the process dynamics. Control laws based on "nominal" models may not meet the design specifications and in some cases they might render the closed-loop system unstable.

Consider a linear process described by

$$A_0\bar{y} = B_0\bar{u} + C_0\bar{d} \tag{5-22}$$

and the nominal model (5-1) approximating (5-22)

$$A\bar{y} = B\bar{u} + C\bar{d}$$

Also let $G_0 = \frac{B_0}{A_0}$, $G = \frac{B}{A}$ represent the true and nominal transfer functions respectively, and define the additive (absolute) model uncertainty by

$$\Delta G = G_0 - G$$

and the multiplicative (relative) by

$$\overline{\Delta G} = \frac{G_0 - G}{G}$$

We shall find conditions involving the model uncertainty and the location of the closed-loop poles so that the MDC closed-loop system is robustly stable.

Theorem 5.1

Let $\bar{p}(\omega)$ be an upper bound on $|\bar{\Delta G}(z)|$, i.e.

$$|\bar{\Delta G}(e^{j\omega})| < \bar{p}(\omega) < \infty$$

Then the closed-loop system under MDC is stable if the following are true:

(i) $A(z)$ and $A_0(z)$ have the same number of zeroes outside the unit circle

(ii) $$|g(e^{j\omega})| < \frac{1-k}{\bar{p}(\omega)} \qquad 0 < k < 1$$

where

$$g(z) = \frac{P(z)\, B_+(z)}{T(z)}$$

(iii) a. $$|1-g(e^{j\omega})| \left|\frac{1-f}{1-fe^{-j\omega}}\right| < \frac{k}{\bar{p}(\omega)}$$

b. $$\frac{T(1)\, B_0(1)}{T_0(1)\, B_+(1)} > 0$$

where

$$T_0 = A_0QB_- + B_0P \tag{5-23}$$

Proof. Theorem 5.2 will be proved in two parts. First, it will be shown that conditions (i) and (ii) result to a stable input-output equation between $\bar{y}$ and $\bar{v}$, and then IMC robustness results and condition (iii) are utilized to show overall stability. From the definition of the generalized input (5-3) and the true model (5-22) we obtain the closed-loop equations between $\bar{y}$ and $\bar{v}$ and $\bar{u}$ and $\bar{v}$

$$T_0 y = B_0 v + QB_- C_0 \bar{d} \tag{5-24}$$

$$T_0 u = A_0 dv - PC_0 \bar{d} \tag{5-25}$$

where T_0 is the actual closed-loop polynomial and is given by (5-23).

Recall that P, Q satisfy

$$T = A\,Q + B_+ P \tag{5-26}$$

and TB_- is the "nominal" closed-loop polynomial.

From (5-25) and (5-26) we obtain, after eliminating QB_-

$$\frac{A}{TB_-}\frac{T_0}{A_0} = 1 + \frac{PB_+}{T}\,\overline{\Delta G} \tag{5-27}$$

Condition (ii) gives

$$\left|\frac{PB_+}{T}\,\overline{\Delta G}\right| \leq \left|\frac{PB_+}{T}\right| \bar{\rho}(\omega) < 1-k \quad \text{for} \quad z = e^{j\omega}$$

which implies that the frequency response of $\frac{PB_+}{T}\Delta G$ does not encircle the point $(-1, j_0)$. Then, it follows from the Nyquist criterion that $\frac{AT_0}{A_0TB_-}$ has an equal number of poles and zeroes outside the unit circle. The number of poles outside the unit circle are the zeroes of A_0 which from condition (i) are equal to the zeroes of A outside the unit circle. This implies that AT_0 can have no other zeroes outside the unit circle except those of A. In other words T_0 has no zeroes outside $|z| = 1$.

For the next step consider the actual system

$$T_0\bar{y} = B_0\bar{v} + QB_-C_0\bar{d} \tag{5-28}$$

the nominal model

$$T\bar{y} = B_+\bar{v} + QC\bar{d} \tag{5-29}$$

and the IMC law

$$\bar{v} = \frac{TF}{(1-B_+F)}\,[\bar{y}_{sp}-y]$$

Define the transfer functions

$$M_0 = \frac{B_0}{T_0}$$

$$M = \frac{B_+}{T} = \frac{B}{TB_-}$$

and the modeling error

$$\overline{\Delta M} = \frac{M_0 - M}{M} \tag{5-30}$$

To invoke the IMC robustness analysis we have to know bounds for $\overline{\Delta M}$. In the following $\overline{\Delta M}$ is related to the known $\bar{p}(\omega)$.

Using (5-26)-(5-30) we can show that

$$\overline{\Delta M} = (1-g)\,\frac{\overline{\Delta G}}{1+(PB_+\overline{\Delta G})/T}$$

or

$$|\overline{\Delta M}(z)| \le |1-g(z)|\,\frac{\bar{p}(\omega)}{\left|1+\dfrac{P(z)B_+(z)}{T(z)}\,\overline{\Delta G}(z)\right|} \quad \text{at} \quad z = e^{j\omega} \tag{5-31}$$

Condition (ii) implies that

$$\left|1+\frac{P(z)B_+(z)}{T(z)}\,\overline{\Delta G}(z)\right| > k \quad \text{at} \quad z = e^{j\omega}$$

and substitution of the above to (5-31) gives

$$|\overline{\Delta M}(z)| < \bar{s}(\omega) \quad \text{at} \quad z = e^{j\omega} \tag{5-32}$$

where

$$\bar{s}(\omega) = \frac{1}{k}\,|1-g(e^{j\omega})|\;\bar{p}(\omega) \tag{5-33}$$

Now, note that both M_0 and M are stable transfer functions. Then as shown in Appendix A, the IMC law for the system (5-28) is robustly stable if $M(1)\,M_0(1) > 0$ and

$$\left|\frac{1-f}{1-fz^{-1}}\right| \leq \frac{1}{S(\omega)}$$

which are both true in view of (5-33) and condition (iii).

Then, the closed-loop system is stable and y, v are bounded. This in view of (5-24) and (5-25) implies that the overall closed-loop system is stable.

Remark 5.1

Condition (iii.b) can be substituted by

$$G(1) \;/\; G_0(1) > 0 \qquad (5\text{-}34)$$

i.e. the condition for robustness of IMC for the system (5-22) (see Theorem A.1).

Proof. From the bound on $|\overline{\Delta G}|$ and condition (ii) of Theorem 5.1 we have

$$\left|\frac{PB_+}{T}\,\overline{\Delta G}\right|_{z=1} < 1$$

which if used in (5-27) gives

$$\frac{A(1)}{A_0(1)}\,\frac{T_0(1)}{T(1)B_-(1)} > 0$$

Multiplying the above by (5-34) we obtain condition (iii.b).

To complete the robustness discussion we show that for open loop stable systems there exist families of polynomials T(z;r) and of filters F(f;z) that satisfy conditions (ii) and (iii) of Theorem 5.1 for all finite $\bar{p}(\omega)$. To do so we need some properties of the MDC pole placement algorithm, which are presented in Appendix C. It is shown there that

$$P(z;r) = \left(1 - \frac{r}{p_{max}}\right) \bar{P}(z;r) \tag{5-35}$$

where $\bar{P}(z;r)$ is a polynomial in z^{-1} and r.

Theorem 5.2

If the system and the nominal system model satisfy the following conditions:

1. both system and model are stable
2. the model uncertainty is bounded, i.e.

   $\bar{p}(\omega) < M$
3. the model steady-state gain has the correct sign, i.e.

   $G(1)G_0(1) > 0$

then the MDC law is robustly stable in the sense that there exists r_{min} and f_{min} such that $\forall$ r $(r_{min}<r\leq 1)$ and $\forall$ f $(f_{min}<f<1)$ the closed-loop system is stable.

Proof. To prove the theorem, we must first show that there exists r_{min} such that $\forall$ r $(r_{min}<r\leq 1)$ the condition (ii) of Theorem 5.1 is satisfied.

To show the above, it suffices to prove that there exists r_{min} such that

$$\phi(\omega,r) = \frac{|T(r,e^{j\omega})|^2}{|B_+(e^{j\omega})|^2} - \frac{\bar{p}^2(\omega)}{(1-k)^2}\,|P(r,e^{j\omega})|^2 > 0 \qquad (5\text{-}36)$$

for every r $(r_{min}<r\leq 1)$.

From (5-35) and the fact that the model is stable ($P_{max} = 1$)

$$|P(r,e^{j\omega})| = (1-r)\,|\bar{P}(r,e^{j\omega})|$$

and using the above in (5-36), we obtain

$$\phi(\omega,r) = \frac{|T(r,e^{j\omega})|^2}{|B_+(e^{j\omega})|^2} - \frac{(1-r)^2}{(1-k)^2}\,\bar{p}^2(\omega)\,|\bar{P}(r,e^{j\omega})|^2$$

Now, $\phi(\omega,r)$ is for every ω a polynomial in r satisfying

$$\phi(\omega,1) > 0$$

since by condition (2) $\rho(\omega) < M$.

Define $r^*(\omega)$ as the maximum real root of $\phi(\omega,r^*) = 0$ in $[0,1)$. If for some ω no such root exists take $r^*(\omega) = 0$.

Then

$$\phi(\omega,r) > 0 \qquad \forall \quad r \quad r^*(\omega)<r\leq 1$$

If we choose

$$r_{min} = \max_{\omega} [r^*(\omega)]$$

then

$$\phi(\omega,r) > 0 \quad \forall \; r \quad r_{min}<r\leq 1$$
$$\text{and} \quad \forall \; \omega \quad 0\leq\omega\leq\Pi$$

and condition (ii) of Theorem 5.1 is satisfied. Therefore T_0 is stable.

Next we show that there exists f_{min} such that $\forall$ f $(f_{min}<f<1)$ condition (iii) of Theorem 5.1 is satisfied.

First, note that condition (3) of Theorem 5.2 and Remark 5.1 prove condition (iii.b). Then the rest of the proof is the same as in Theorem A.1.

CHAPTER VI
ADAPTIVE MODEL DEPENDENT CONTROL

Introduction

Adaptive controllers based on the Model Dependent Control design method are presented in this chapter. MDC combines a pole/zero placement design method for regulation and the Internal Model Control approach for set-point following. The closed-loop performance depends on two parameters, one determining the closed-loop poles for disturbance rejection and the second the closed-loop poles for set-point tracking.

Adaptive MDC belongs to the class of indirect adaptive controllers (Goodwin and Sin, 1984, p. 181) where the model parameters are estimated on-line, and the control law is updated using the current estimates. Stability and convergence analysis for indirect adaptive control has been a real challenge in recent years. The main reason is that the on-line identified model may not be stabilizable; i.e. an unstable pole-zero cancellation may occur.

Many methods have been proposed to overcome this problem. One way is to restrict the parameter estimates in a region where the model is always stabilizable (Goodwin and Sin, 1981; De Larminat, 1984; Kreisselmeier, 1985). The persistency of excitation condition was used by Elliot et al. (1985), and Anderson and Johnstone (1985) to establish parameter convergence to their true values (it is assumed that the system is stabilizable) and thus avoiding the pole-zero cancellation. A method

which alleviates the need for external exciting inputs was proposed by Leal and Goodwin (1984). The idea is to modify the estimates before using them so as to avoid pole-zero cancellations and yet retain the other estimator properties. Note that all these methods affect only the estimator and are independent of the control design method. Any of these methods can be used in the adaptive MDC framework since all of them satisfy the estimator properties required to prove MDC stability and convergence.

Given that the identified model is always stabilizable, the stability and convergence of many indirect adaptive schemes has been established. The asymptotic stability of pole-placement algorithms has been proved in Goodwin et al. (1984), Elliot et al. (1985, and Anderson and Johnstone (1985), while Samson et al. (1981) and Kreisselmeier (1985), showed the convergence of adaptive optimal control methods. It should also be mentioned that recently Ossman and Kamen (1986) and Ossman (1986) developed and proved convergence for an adaptive optimal control algorithm that does not require pointwise stabilizability of the estimated model. Instead they use an estimator that forces the estimates to converge to a stabilizable region.

All control algorithms described above require knowledge of the exact system order (and in some cases of the time delay) for the following reason: if the order is overestimated the model is not stabilizable and if underestimated the estimator does not converge. Not such an assumption is made here. The only a priori knowledge required is an upper bound of the process order (which is a minimal requirement for adaptive control), and some limited information about the process gain.

Two different adaptive MDC algorithms are developed next. For the first one, the parameters that specify the location of the closed-loop poles are constant. This algorithm is destined for application to systems that are essentially linear and will be referred to as Fixed Adaptive MDC (FAMDC). The second algorithm is developed with non-ideal situations in mind like those described in Chapters II, III, and V. For this case the two MDC parameters are tuned on-line depending on a measure of the plant/estimated model mismatch and the control algorithm will be referred to as Cautious Adaptive MDC (CAMDC). This can be viewed as an extension of the cautious controller design method presented in Chapters III and V that allows non-minimum phase and/or open-loop unstable systems to be treated.

Adaptive MDC can be recast into a pole-zero placement algorithm but with an important difference from other similar controllers: the target closed-loop poles are time varying. For this reason the stability and convergence of the MDC cannot be deduced from already existing results. MDC stability and convergence are examined in this chapter combining linear time varying operator and estimator properties.

The performance of both algorithms is demonstrated by simulations on the continuous stirred tank reactor presented in Chapter III.

Adaptive MDC Law

It is assumed that the process to be controlled is described by

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t) + d(t) \qquad (6\text{-}1)$$

where $y(t)$ is the output, $u(t)$ is the input, $A(q^{-1})$, $B(q^{-1})$ are polynomials in the backwards shift operator q^{-1} given by

$$A(q^{-1}) = 1 - a_1 q^{-1} - a_2 q^{-2} - \ldots - a_n q^{-n} \tag{6-2}$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_1 q^{-l} \; ; \quad l \leq n \tag{6-3}$$

and d(t) accounts for disturbances that without loss of generality may be considered as the sum of a constant and a time varying term

$$d(t) = d + \bar{d}(t) \tag{6-4}$$

The following assumptions are made for the process (6-1)

A1. An upper bound n of the system order is known.

A2. The system is stabilizable, i.e. $A(q^{-1})$ and $B(q^{-1})$ do not have a common unstable zero.

It is not assumed here that the leading coefficients of $B(q^{-1})$ are nonzero, thus allowing for an unknown time delay. However, it is assumed that the sign of B(1) as well as a lower bound of its magnitude are known, i.e.

A3. sign B(1) is known and

$$|B(1)| \geq b_{min}$$

A4. The disturbances are asymptotically time invariant, i.e.

$$\lim_{t \to \infty} \bar{d}(t) = 0$$

and

$$\sum_{i=1}^{\infty} \bar{d}^2(i) < \infty$$

Rewrite (6-1) as

$$y(t) = x^T(t-1)\theta + \bar{d}(t) \tag{6-5}$$

where

$$x^T(t-1) = [y(t-1) \ \ldots \ y(t-n) \ u(t-1) \ \ldots \ u(t-l) \ 1]^T$$

$$\theta = [a_1 \ \ldots \ a_n \ b_1 \ \ldots \ b_l \ d]^T$$

The unknown parameters are estimated on-line and using the estimates at time t we form the time varying operators

$$\hat{A}(t,q^{-1}) = 1 - \hat{a}_1(t)q^{-1} - \hat{a}_2(t)\,q^{-2} - \ldots - \hat{a}_n(t)q^{-n} \tag{6-6}$$

$$\hat{B}'(t,q^{-1}) = \hat{b}_1(t)\,q^{-1} + \hat{b}_2(t)q^{-2} + \ldots + \hat{b}_l(t)q^{-l} \tag{6-7}$$

where $\hat{a}_i(t)$ (i=1,...,n), $\hat{b}_j(t)$ (j=1,...,l), and $\hat{d}(t)$ are the estimated parameters at time t.

The MDC law requires factorization of the polynomial $\hat{B}'(t,q^{-1})$. This factorization is ill-conditioned whenever the leading coefficients of $\hat{B}(t,q^{-1})$ become very small and may result to unbounded control law operators. To overcome the problem leading coefficients absolutely smaller than a prescribed number are discarded, and $\hat{B}'(t,q^{-1})$ is reduced to the following well-conditioned for factorization polynomial

$$\hat{B}(t,q^{-1}) = \hat{b}_{k(t)}(t)q^{-k(t)} + \ldots + \hat{b}_{m(t)}(t)q^{-m(t)} \quad 1 \le k(t) \le m(t) \le l \tag{6-8}$$

with

$$|\hat{b}_{k(t)}(t)| \ge \varepsilon_1 \ , \quad |\hat{b}_{m(t)}| \ge \varepsilon_1$$

and

$$|\hat{b}_j(t)| < \varepsilon_1 \qquad j = 1, \ldots, k(t)-1, m(t)+1, \ldots, l$$

Note that small trailing coefficients of $\hat{B}'(t,q^{-1})$ are also discarded, and this may reduce significantly the factorization calculations. This is equivalent to taking all zeroes of $\hat{B}'(t,q^{-1})$ close to the origin equal to zero.

The adaptive MDC laws are then obtained using the algorithm described in section 5.4 by substituting at each sampling interval the unknown polynomials $A(q^{-1})$, $B(q^{-1})$ with the estimated $\hat{A}(t,q^{-1})$, $\hat{B}(t,q^{-1})$, respectively. Since there are some differences in the adaptive case the algorithm is repeated here:

0. Choose a small positive $\varepsilon_1 < b_{min}/l$
1. Find $p_{max}(t)$ from

$$p_{max}(t) = \max_i \{|\lambda_i(t)| + \varepsilon_2, 1\} \qquad i = 1, \ldots, n \; ; \quad \varepsilon_2 \geq 0$$

where $\lambda_i(t)$ is the ith zero of $\hat{A}(t,z^{-1}) = 0$, and

$$\varepsilon_2 \begin{cases} = 0 & \text{if } \lambda_{max} < 1 - \sigma \; ; \quad \sigma > 0 \\ > \dfrac{\sigma\lambda_{max}}{1-\sigma} & \text{if } \lambda_{max} \geq 1 - \sigma \end{cases}$$

$$\lambda_{max} = \max_i \{|\lambda_i(t)|\}$$

The above choise for ε_2 guarantees that

$$\frac{\lambda_{max}}{p_{max}} < 1 - \sigma \tag{6-9}$$

It is noted that λ_{max} can be obtained numerically without factoring $\hat{A}(t,q^{-1})$.

2. Factor $\hat{B}(t,q^{-1})$ as

$$\hat{B}(t,q^{-1}) = \hat{B}_-(t,q^{-1})\ \hat{B}_+(t,q^{-1}) \tag{6-10}$$

with

$$\hat{B}(t,q^{-1}) = \beta(t) \prod_i (1-z_i^-(t)q^{-1}) \qquad i = 1, \ldots, l_1(t) \tag{6-11}$$

$$\beta(t) = (-1)^{l_1(t)}\ \hat{b}_{m(t)}(t) \prod_i \frac{1}{z_i^-(t)} \tag{6-12}$$

$$\hat{B}_+(t,q^{-1}) = q^{-k(t)} \prod_j (-\frac{1}{z_j^+} + q^{-1}) \qquad j = 1, \ldots, l_2(t) \tag{6-13}$$

$$l_1(t) + l_2(t) = m(t) - k(t)$$

and form

$$\hat{\bar{B}}_+(t,q^{-1}) = \prod_i (-\bar{z}_i^+(t) + q^{-1}) \qquad i = 1, .., l_3(t) \tag{6-14}$$

$$l_3(t) \le l_2(t)$$

where

$$|z_i^-| < 1 - \sigma \qquad 0 < \sigma < 1$$

$$|z_i^+| \ge 1 - \sigma$$

and $|z_1^+|$ denotes those z_1^+ with $|z_1^+| > 1$. Note that σ is usually a small positive number.

The factorization employed here is different from the factorization presented in section 5.4. The reason is explained at the end of this section in Remarks 6.1 and 6.2.

3. Form

$$\hat{F}_D(t,q^{-1}) = (1-\hat{f}(t)q^{-1})\,\hat{B}_+(t,q^{-1}) \tag{6-15}$$

$$F_N(t) = 1-f(t)$$

$$\hat{C}(t,q^{-1}) = \hat{A}(t,q^{-1})\,\hat{F}_D(t,q^{-1}) \tag{6-16}$$

$$= 1 + \hat{C}_1(t)q^{-1} + \ldots + \hat{C}_{nc}(t)q^{-n_c}$$

and $$n_c(t) = n + l_3(t) + 1$$

4. Form the polynomial

$$\hat{T}(t,q^{-1}) = 1 + \hat{C}_1(t)\frac{r}{p_{max}(t)}q^{-1} + \ldots + \hat{C}_{n_c}(t)\left(\frac{r}{p_{max}(t)}\right)^{n_c(t)} q^{-n_c(t)} \tag{6-17}$$

and solve the Bezoutian

$$\hat{T}(t,q^{-1}) = \hat{C}(t,q^{-1})\hat{Q}_1(t,q^{-1}) + \hat{B}_+(t,q^{-1})\hat{P}(t,q^{-1}) \tag{6-18}$$

to find the polynomials $\hat{Q}_1(t,q^{-1})$ and $\hat{P}(t,q^{-1})$, where

$$\deg(\hat{P}(t,q^{-1}) = n_c(t) - 1$$

$$\deg(\hat{Q}_1(t,q^{-1}) = k(t) + l_2(t) - 1$$

5. Form the operator

$$\hat{K}(t,q^{-1}) = \hat{F}_D(t,q^{-1})\,[\hat{Q}_1(t,q^{-1})\hat{A}(t,q^{-1})F_N(t) + \hat{P}(t,q^{-1})] \qquad (6\text{-}19)$$

and check if the zeroes of

$$\hat{T}_c(t,q^{-1}) = \hat{B}_-(t,q^{-1})\hat{F}_D(t,q^{-1})\hat{T}(t,q^{-1}) + [\hat{B}'(t,q^{-1}) - \hat{B}(t,q^{-1})]\hat{K}(t,q^{-1}) \qquad (6\text{-}20)$$

are inside the unit circle. If not reduce ε_1 by taking

$$\varepsilon_1 = \max_j |\hat{b}_j(T)| \qquad j = 1, \ldots, k(t)-1, m(t)+1, \ldots, l \qquad (6\text{-}21)$$

so as to include more terms in $\hat{B}(t,q^{-1})$ and go to step 2.
As shown later this step is crucial in proving the stability of the MDC law. At most l iterations are required to find a $\hat{B}(t,q^{-1})$ such that $\hat{T}(t,q^{-1})$ is stable (for at most l iterations $\hat{B}' = \hat{B}$ and $\hat{T}_c = \hat{B}_- \hat{F}_D \hat{T}$). See Remark 6.2 below for some implementation aspects.

6. Obtain the MDC law from

$$\hat{H}(t,q^{-1})u(t) = -\hat{K}(t,q^{-1})y(t) + \hat{R}(t,q^{-1})y_{sp}(t) \qquad (6\text{-}22)$$

where

$$\hat{H}(t,q^{-1}) = \hat{Q}(t,q^{-1})\hat{B}_-(t,q^{-1})[\hat{F}_D(t,q^{-1}) - \hat{B}_+(t,q^{-1})F_N(t)] \qquad (6\text{-}23)$$

$$\hat{Q}(t,q^{-1}) = \hat{Q}_1(t,q^{-1})\hat{F}_D(t,q^{-1}) \qquad (6\text{-}24)$$

$$\hat{R}(t,q^{-1}) = F_N(t)\hat{T}(t,q^{-1}) \qquad (6\text{-}25)$$

In the case of FAMDC the parameters f and r are prespecified constants and F_N is a constant. In the case of CAMDC they are time varying as prescribed next in section 6.3.

Remark 6.1

The factorization (6-10)-(6-14) ensures that $\hat{B}_+(t,q^{-1})$ and $\hat{\bar{B}}_+(t,q^{-1})$ have bounded coefficients since

$$0 < \frac{1}{z_1^+(t)} \leq \frac{1}{1-\sigma}$$

In addition, suppose that the coefficients of $\hat{B}'(t,q^{-1})$ are bounded and that $\varepsilon_1 \geq \varepsilon_{1,\min} > 0$. Then $\hat{B}_-(t,q^{-1})$ has bounded coefficients and its leading coefficient $\beta(t)$ is bounded away from zero.

Proof. First note that

$$\prod_i z_i^-(t) \prod_j z_j^+(t) = \frac{\hat{b}_{m(t)}(t)}{\hat{b}_{k(t)}(t)}$$

and since $\varepsilon_{1,\min} \leq \varepsilon_1 < b_{\min}/1$.

$$0 < \left| \frac{\hat{b}_{m(t)}(t)}{\hat{b}_{k(t)}(t)} \right| < \infty$$

The last inequality means that the magnitude of the zeroes of $\hat{B}(t,q^{-1})$ is bounded away from zero and infinity, i.e. for every ε_1 there exists ε_3 such that

$$0 < \varepsilon_3 \leq |z_1^+| < 1 \tag{6-26}$$

$$1 - \sigma \leq |z_1^+| < \frac{1}{\varepsilon_3} < \infty \tag{6-27}$$

and

$$\varepsilon_1 \leq |\beta(t)| \leq \frac{M}{\varepsilon_3 l_1(t)} < \infty \tag{6-28}$$

where M is an upper bound on the coefficients of $\hat{B}'(t,q^{-1})$.

Then follows from (6-11) and Lemma D.2(i) that $\hat{B}_-(t,q^{-1})$ is bounded.

Remark 6.2

Step 5 of the adaptive MDC algorithm requires to check the stability of $\hat{T}_c(t)$. If ε_1 is chosen sufficiently small, one iteration of the algorithm will suffice in most cases in practice. In addition, it is not required to utilize a necessary and sufficient condition for stability (e.g. the Jury test). Instead a much simpler sufficient condition derived below may be used.

From (6-20), dropping the arguments t and q^{-1}

$$\hat{T}_c = \hat{B}_- \hat{F}_D \hat{T} + \tilde{B} \hat{K} \tag{6-29}$$

where

$$\tilde{B} = \hat{B}' - \hat{B} \tag{6-30}$$

In view of (6-19)

$$\hat{K} = \hat{F}_D \hat{K}_1$$

and substituting the above in (6-29) we obtain

$$\hat{T}_c = \hat{B}_- \hat{F}_D \hat{T} \left(1 + \frac{\hat{K}_1}{\hat{B}_- \hat{T}} \tilde{B}\right) \tag{6-31}$$

Then a sufficient condition for stability is given by

$$\frac{|\hat{K}_1(z^{-1})|}{|\hat{B}_-(z^{-1})T(z^{-1})|} \leq \frac{1}{|\tilde{B}(z^{-1})|} \quad \text{for} \quad z = e^{j\omega} \quad 0 \leq \omega \leq \Pi \tag{6-32}$$

where z substitutes q and the argument t is omitted. Note that

$$|\hat{K}_1(z^{-1})| \leq \sum_i |\hat{k}_{1i}| \tag{6-33}$$

where $\hat{k}_{1i}$ denotes the ith coefficient of $\hat{K}_1(z^{-1})$. Also

$$|B(z^{-1})| \leq \tilde{l}\, \varepsilon_1 \qquad \tilde{l} = 1 + k(t) - m(t) - 1 \tag{6-34}$$

Let

$$\hat{d}_{max} = \sum_i |\hat{k}_{ji}|$$

and

$$\hat{d}_{min} = \min_{\omega} |\hat{B}_-(e^{-j\omega})\, \hat{T}(e^{-j\omega})| \qquad 0 \leq \omega \leq \Pi \tag{6-35}$$

Using (6-33)-(6-35) we can easily show that (6-32) is satisfied if the following is true

$$\frac{\hat{d}_{max}}{\hat{d}_{min}} \leq \frac{1}{\tilde{l}\ \varepsilon_1} \tag{6-36}$$

It can be shown that

$$\hat{d}_{min} = \min\ [|\hat{B}_-(1)\ \tilde{T}(1)|,\ |\hat{B}_-(-1)\ \tilde{T}(-1)|] \tag{6-37}$$

Then, (6-36) and (6-37) constitute an easily implementable sufficient condition for the stability of $\tilde{T}_c(t,q^{-1})$.

It is proved in Lemma D.3 that there exists an $\varepsilon_{1,max}$ greater than zero such that (6-36) is satisfied if $\varepsilon_1 = \varepsilon_{1,max}$ for every time.

Cautious Adaptive MDC Tuning

The adaptive controller is based on a linear time invariant process model identified on-line. If the actual process is time varying and/or nonlinear, the estimated model may describe the process poorly for the reasons explained in Chapters II to IV.

Since in the adaptive case the model uncertainty is varying, the MDC tuning parameters should be varying too. For simplicity in tuning, the CAMDC is designed to have only one free parameter by taking

$$f(t) = \min\ [r(t),\ 1 - \sigma]$$

Following the same guidelines as in Chapters II to IV, if the model uncertainty is large r(t) takes values close to one and leads to a conservative but robust control action at least for open-loop stable systems. As the model improves r(t) decreases to speed up response.

Specifically, r(t) is given by

$$r(t) = pr(t-1) + (1-p)\mu(t) \qquad 0 \leq p < 1 \tag{6-38}$$

Here

$$\mu(t) = r_{min} + [1-r_{min}]\mu_1(t)$$

with

$$\mu_1(t) = \frac{\rho(t)}{\delta|\hat{b}_{k(t)}(t)| + \rho(t)}$$

$$0 \leq r_{min} < 1$$

where $\rho(t) \geq 0$ is a measure of the plant/model mismatch obtained from the estimator. Note that

$$r_{min} \leq r(t) < 1$$

and for a perfect model ($\rho(t) = 0$) $r(t) = r_{min}$.

The measure of the plant/model mismatch is chosen as

$$\rho(t) = \frac{|\varepsilon(t)|}{[\lambda(t) + w(t-1)]^{1/2}}$$

where $\varepsilon(t)$ is the prediction error, and $\lambda(t)$, λ_{min}, $w(t-1)$ are estimator quantities defined in Appendix B. It is proved in Appendix B that for linear deterministic systems $\lim_{t\to\infty} \rho(t) = 0$ and consequently

$$\lim_{t\to\infty} r(t) = r_{min} \tag{6-39}$$

which is a key property in establishing the controller stability as shown next.

Stability and Convergence Analysis of Fixed Adaptive MDC and Cautious Adaptive MDC

The stability of the algorithms will be studied using the closed-loop equations relating the process inputs and outputs. It is obvious from (6-1) and (6-13) that these equations involve time varying operators. Hence, it is not possible to obtain the closed-loop equations by direct manipulation of (6-1) and (6-13). The problem is that the convolution of time varying operators does not commute, e.g.

$$H(t,q^{-1}) \cdot B(t,q^{-1}) \neq B(t,q^{-1}) \cdot H(t,q^{-1})$$

Instead, the operator product (as a polynomic product) commutes, e.g.

$$H(t,q^{-1})\, B(t,q^{-1}) = B(t,q^{-1})\, H(t,q^{-1})$$

In Appendix D, some properties of linear time varying operators are derived and then used to obtain the closed-loop system equations.

The aim is to express the closed-loop behavior in terms of known time varying operators and quantities depending on estimator properties, which are next invoked to prove stability.

A minimal set of properties that the estimator should satisfy in order to prove stability is given next.

Estimator Properties

It is assumed here that the estimator has the following properties:

E1. The parameter estimates $\hat{\theta}(t)$ are bounded, i.e. $|\hat{\theta}(t)| < M$

E2. $\lim_{t\to\infty} |\hat{\theta}(t) - \hat{\theta}(t-i)| = 0 \qquad 0 \leq i < \infty$

E3. $|\varepsilon(t)| \leq \delta_1(t) + \delta_2(t)\,|x(t-1)|$

$|\eta(t)| \leq \delta_1(t)$

where

$\varepsilon(t) = y(t) - x^T(t-1)\,\hat{\theta}(t-1)$

is the a priori prediction error,

$\eta(t) = y(t) - x^T(t-1)\,\hat{\theta}(t)$ (6-40)

is the aposteriori prediction error, and

$0 \leq \delta_2(t) < \infty\,, \quad \lim_{t\to\infty} \delta_2(t) = 0$

$0 \leq \delta_1(t) < \infty$

Since d(t) is a asymptotically invariant

$\lim_{t\to\infty} \delta_1(t) = 0$

and

$\lim_{t\to\infty} \eta(t) = 0$

E4. The estimator ensures that $\hat{B}_+(t)$ and A(t) are coprime for any time t (pointwise stabilizability).

E5. The estimator ensures that $\hat{B}'(1)$ has the right sign and that $|\hat{B}'(1)| \geq b_{min}$

E6. The parameter estimates converge, i.e.

$\lim_{t\to\infty} \hat{\theta}(t) = \hat{\theta}_\infty$

Note that $\hat{\theta}_\infty$ is not necessarily equal to the true parameter vector θ.

An estimator that satisfies all the above properties is given in Appendix B. It is based on the least squares estimator described in Leal and Goodwin (1984) which satisfies E1-E4 and E6. A modification described in Goodwin and Sin (1984, p. 91) is also employed so that the estimator satisfies in addition the parameter constraints of E5.

The stability and convergence analysis that follows is not restricted to any particular estimation scheme but is valid for any estimator satisfying E1-E6.

The Time Varying Closed-Loop System

It is proved in Appendix D that the closed-loop system satisfies the equations given next in Lemma 6.1. Henceforth q^{-1} is suppressed.

Lemma 6.1

The closed-loop dynamics for the fixed adaptive and cautious adaptive MDC are described by

$$\hat{T}_c(t)\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \hat{B}(t) \\ \hat{A}(t) \end{bmatrix}\left[\hat{R}(t)y_{sp}(t)\right] + \begin{bmatrix} \hat{H}(t) \\ -\hat{K}(t) \end{bmatrix}\eta(t) + \begin{bmatrix} \hat{H}(t) \\ -\hat{K}(t) \end{bmatrix}\hat{d}(t)$$

$$\begin{bmatrix} \Delta\hat{B}\hat{K}(t) + \Delta\hat{H}\hat{A}(t) & \Delta\hat{B}\hat{H}(t) - \Delta\hat{H}\hat{B}(t) \\ \Delta\hat{A}\hat{K}(t) - \Delta\hat{K}\hat{A}(t) & \Delta\hat{A}\hat{H}(t) + \Delta\hat{K}\hat{B}(t) \end{bmatrix}\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \qquad (6\text{-}41)$$

where $\Delta\cdot\cdot(T)$ represents operators defined from (D-13) in Appendix D, and $\hat{T}_c(t)$ is given by (6-20).

For convenience we rewrite (6-41) as

$$\hat{T}_c(t) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} l_1(t) \\ l_2(t) \end{bmatrix} \tag{6-42}$$

where the definition of $l_1(t)$ and $l_2(t)$ is obvious upon inspection of (6-41) and (6-42).

To prove stability from (6-21), we will need the following Lemmas.

Lemma 6.2

The closed-loop polynomial

$$\hat{T}_c(t,q^{-1}) = \tau_0(t) + \tau_1(t)q^{-1} + \ldots + \tau_{n_T}(t)q^{-n_T} \tag{6-43}$$

(with n_T being the maximal order of $\hat{T}_c(t)$ for all times) has the following properties:

i. $|\tau_0(t)| \geq \varepsilon_4 > 0$

ii. The coefficients $\tau_i(t)$ ($i = 0, \ldots, n_T$) are bounded and satisfy

$$\lim_{t\to\infty} \left| \frac{\tau_i(t)}{\tau_0(t)} - \frac{\tau_i(t-j)}{\tau_0(t-j)} \right| = 0 \qquad 0 \leq j < \infty \tag{6-44}$$

iii. For all times t, all zeroes of $\hat{T}_c(t,z^{-1})$ lie inside a circle in the complex plane with radius $1-\sigma$; in other words $\hat{T}_c(t,q^{-1})$ is pointwise stable.

Proof.

i,ii. These properties are consequence of the estimator properties E1-E6 and follow easily from (6-20) and Lemmas D2, D3 of Appendix D. A formal proof is given in Appendix D.

iii. This is true by construction (step 5 of the adaptive MDC algorithm).

Lemma 6.3

The scalar system

$$\hat{T}_c(t,q^{-1})W(t) = l(t) \tag{6-45}$$

is exponentially stable after some finite time to, and satisfies

$$|W(t)| \leq C_1(t) + C_2||l(t)||_\infty \qquad t \geq t_0 \tag{6-46}$$

where

$$||l(t)||_\infty = 0 \overset{\max}{\leq i \leq t} |l(i)|$$

and

$$0 \leq C_1(t) < \infty\ , \quad \lim_{t\to\infty} C_1(t) = 0$$

$$0 < C_2 < \infty$$

Proof. The system defined from (6-43) and (6-45) admits the following state-space representation in the controllable canonical form

$$Z(t+1) = M(t)Z(t) + N(t)l(t) \tag{6-47}$$

$$W(t) = C\,Z(t+1) \tag{6-48}$$

where

$$M(t) = \begin{bmatrix} -\frac{\tau_1(t)}{\tau_0(t)} & -\frac{\tau_2(t)}{\tau_0(t)} & \cdots & -\frac{\tau_{n_T}(t)}{\tau_0(t)} \\ 1 & 0 & & \vdots \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & \cdots & 1\ 0 \end{bmatrix} \tag{6-49}$$

$$Z(t) = [W(t-1)W(t-2) \ \ldots \ W(t-n_T)]^T$$

$$N(t) = \left[\frac{1}{\tau_0(t)}\ 0 \ \ldots \ 0\right]^T$$

$$C = [1\ 0 \ \ldots \ 0]^T$$

Lemma 6.2 implies that the time varying matrix M(t) satisfies

$$||M(t)|| < \infty \qquad \forall\ t \tag{6-50}$$

$$\lim_{t\to\infty} ||M(t+1) - M(t)|| = 0 \tag{6-51}$$

and

$$\max_j |\lambda_j(M(t))| \leq 1 - \sigma < 1\ , \quad \sigma > 0 \tag{6-52}$$

where $||\cdot||$ denotes the induced Euclidean matrix norm, and $\lambda_j(M(t))$ is the jth eigenvalue of M(t). Equation (6-50) follows from property (i) of Lemma 6.2, (6-51) from (6-44), and (6-52) from property (iii) of Lemma 6.2.

It has been proved in Desoer (1970) that if M(t) satisfies (6-50), (6-52) and is slowly time varying in the sense that

$$\sup_i ||M(i+1) - M(i)|| \leq \delta < \infty \tag{6-53}$$

where δ is a constant depending on system parameters, then the autonomous system resulting from (6-47) for $l(t) = 0$ is exponentially stable.

Equation (6-51) implies that there exists a finite time t_0 such that

$$\sup_{t_0 \leq t} ||M(t+1) - M(t)|| \leq \delta$$

Then, the system

$$Z(t+1) = M(t)Z(t) \qquad Z(t_0) = Z_0 \tag{6-54}$$

$$W(t) = C\,Z(t+1) \qquad t \geq t_0$$

is exponentially stable. This implies that the transition matrix of (6-54)

$$\phi(t,t_0) = \prod_{i=t_0}^{t-1} M(i)$$

satisfies

$$||\phi(t,t_0)|| \leq K\lambda^{t-t_0} \qquad 0 < \lambda < 1 \; ; \quad 0 < K < \infty \tag{6-55}$$

Now the input-output stability of the forced system (6-47) can be studied in terms of the transition matrix $\phi(t,t_0)$. Note that exponential stability for a time varying linear system does not in general imply BIBO (or l_∞) stability. Therefore, BIBO stability should be directly proved from (6-55) and (6-47).

The solution of (6-47) is given by

$$Z(t+1) = \phi(t,t_0)Z_0 + \sum_{\tau=t_0}^{t} \phi(t,\tau)N(\tau)l(\tau) \tag{6-56}$$

From property (i) of Lemma 6.2 $|\tau_0(t)| \geq \varepsilon_4 > 0$ and therefore

$$|N(\tau)| \leq N = \frac{1}{\varepsilon_4} < \infty$$

Taking norms on both sides of (6-56) and using (6-48) (note $|C|=1$) we obtain

$$|W(t)| \leq ||\phi(t,t_0)||\,|Z_0| + N\left\{\sum_{\tau=t_0}^{t} ||\phi(t,\tau)||\right\} ||l(t)||_\infty \tag{6-57}$$

Using (6-55) into (6-57)

$$|W(t)| \leq K\lambda^{t-t_0}|Z_0| + NK\left\{\sum_{\tau=t_0}^{t} \lambda^{t-\tau}\right\} ||l(t)||_\infty \tag{6-58}$$

Noting that

$$\sum_{\tau=t_0}^{t} \lambda^{t-\tau} = \sum_{i=0}^{t-t_0} \lambda^i \leq \frac{1}{1-\lambda}$$

and substituting the above into (6-58) we obtain

$$|W(t)| \leq C_1(t) + C_2 \, ||l(t)||_\infty$$

where

$$0 < C_2 = \frac{KN}{1-\lambda} < \infty$$

and

$$0 \leq C_1(t) = K\lambda^{t-t_0} |Z_0)| < \infty$$

Since t_0 is finite, Z_0 is bounded and

$$\lim_{t\to\infty} C_1(t) = 0$$

q.e.d.

Lemma 6.4

If the estimator satisfies the properties E1-E6, then the RHS of (6-41) satisfies

$$|l_1(t)| \leq \gamma_1 + \gamma_2(t) \, [||u(t)||_\infty + ||y(t)||_\infty] \qquad (6\text{-}59)$$

$$|l_2(t)| \leq \delta_1 + \delta_2(t)\,[||u(t)||_\infty + ||y(t)||_\infty] \tag{6-60}$$

where

$$0 < \gamma_1,\ \delta_1 < \infty$$

$$0 \leq \gamma_2(t),\ \delta_2(t) < \infty$$

$$\lim_{t\to\infty} \gamma_2(t) = \lim_{t\to\infty} \delta_2(t) = 0$$

Proof. The Lemma follows easily from estimator properties E1-E4. A formal proof is presented in Appendix D.

Bringing all the pieces together, we formulate the final stability and convergence theorem for the Adaptive MDC algorithms.

Theorem 6.1

If the estimator satisfies E1-E6, then the FAMDC and CAMDC algorithms result to

i. bounded input and output variables, i.e.

$$||u||_\infty < M \tag{6-61}$$

$$||y||_\infty < M$$

where $||\cdot||_\infty = \sup_i ||\cdot(i)||$ is the l_∞ norm

ii. closed-loop performance described by

$$\lim_{t\to\infty}\left[y(t) - \frac{\hat{B}_\infty \hat{R}_\infty}{\hat{T}_{c\infty}} y_{sp}(t)\right] = 0 \qquad (6\text{-}62)$$

Proof.

i. From (6-42) and Lemma 6.3

$$|y(t)| \leq C_1(t) + C_2 \, ||l_1(t)||_\infty$$

$$|u(t)| \leq C_1(t) + C_2 \, ||l_2(t)||_\infty$$

which in view of Lemma 6.4 yield

$$||y(t)||_\infty \leq S_1 + S_2(t) \, ||y(t)||_\infty + S_2(t) \, ||u(t)||_\infty \qquad (6\text{-}63)$$

$$||u(t)||_\infty \leq S_1 + S_2(t) \, ||y(t)||_\infty + S_2(t) \, ||u(t)||_\infty \qquad (6\text{-}64)$$

where

$$0 < S_1 < \infty$$

$$0 \leq S_2(t) < \infty \quad \text{and} \quad \lim_{t\to\infty} S_2(t) = 0$$

Then from (6-63)-(6-64), and the Small Gain Theorem

$$||y||_\infty < \infty$$

$$||u||_\infty < \infty$$

q.e.d.

ii. Since d(t) is bounded and asymptotically constant (see Appendix B) the estimated parameters converge. This implies that in the infinite time limit all time varying operators converge to time invariant ones. Then we obtain from (6-41)

$$\lim_{t\to\infty}\left[y(t) - \frac{\hat{B}_\infty \hat{R}_\infty}{\hat{T}_{C\infty}} y_{sp}(t) - \frac{\hat{H}_\infty}{\hat{T}_\infty} d_\infty\right] = 0 \qquad (6\text{-}65)$$

where the subscript ∞ denotes the invariant operators obtained at the infinite time limit. The last term is zero since $\hat{H}_\infty(q^{-1}=1) = 0$ by design, and (6-62) follows from (6-65).

Example

The FAMDC and the CAMDC performance is demonstrated on the same stirred tank reactor used in Chapter III.

The controller tested here use the UDU factorization of the recursive least squares estimator with variable forgetting factor and parameters as in Table 3.2.

Figures 6-1 to 6-3 examine the performance of FAMDC and Figure 6-4 that of CAMDC with respect to set-point tracking for a sequence of set-point changes. Figure 6-1 presents the performance of FAMDC for f = r = 0 which aims at maximum speed and is equivalent to MV control. Large oscillations in both the output and the control variable are observed as is also the case in Figure 3-1 for the MV control law. More robust solutions are obtained by increasing f and r. Figure 6-2 depicts the

FAMDC performance for f = r= 0.5 and Figure 6-3 for f = r = 0.7. It is clearly seen from these figures that increasing f and r slows down the response but it also eliminates the oscillatory behavior. Small f and r perform well if the plant/model mismatch is small but performance deteriorates as soon as the modeling error increases. This is clearly shown in Figure 6-2 where for some of the set points the controller performs fairly well, but for others (second and fourth) the performance deteriorates due to a poor estimated model. Performance improves as soon as the modeling error drops. It takes an increase of f from 0.5 to 0.7 (r=f) to improve robustness and eliminate oscillations as seen from Figure 6-3. However, this is achieved at the expense of overall performance as seen from the same figure. In Figure 6-4 the performance of the CAMDC for $r_{min} = 0$ is shown. The last plot in this figure is that of f(t) = r(t). It is seen that f(t) increases after each set-point reflecting the increase in modeling error due to the fact that the plant is nonlinear. This way the oscillatory behavior of Figures 6-1 and 6-2 is eliminated. As the modeling error drops, f(t) decreases improving performance.

Figures 6-5 to 6-7 show the FAMDC performance with respect to regulation around the set-point $y_{sp} = 0.05$ and for the same sequence of disturbance changes as in Figure 3-11 for the CSTC. The same behavior as in Figures 6-1 to 6-3 is repeated here. For r = f = 0 (Figure 6-5) fast disturbance rejection is observed at the expense of oscillatory behavior. As r = f increases (Figure 6-6 and 6-7) oscillations are reduced drastically. Finally, Figure 6-8 depicts the CAMDC performance for the

same set of disturbance changes as in Figures 6-5 to 6-7. It is seen again that r(t) = f(t) is adjusted on-line so as to eliminate oscillations when needed and ultimately not to restrict the closed-loop performance.

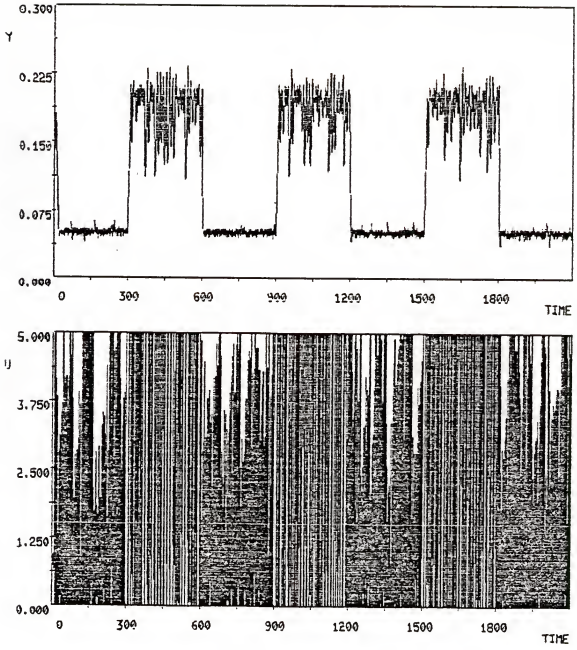


Figure 6-1. FAMDC with f=r=0 for set-point changes.

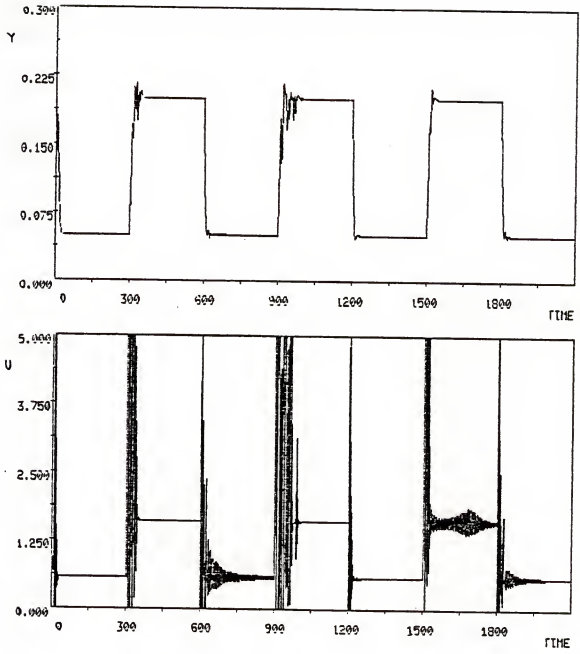


Figure 6-2. FAMDC with f=r=0.5 for set-point changes.

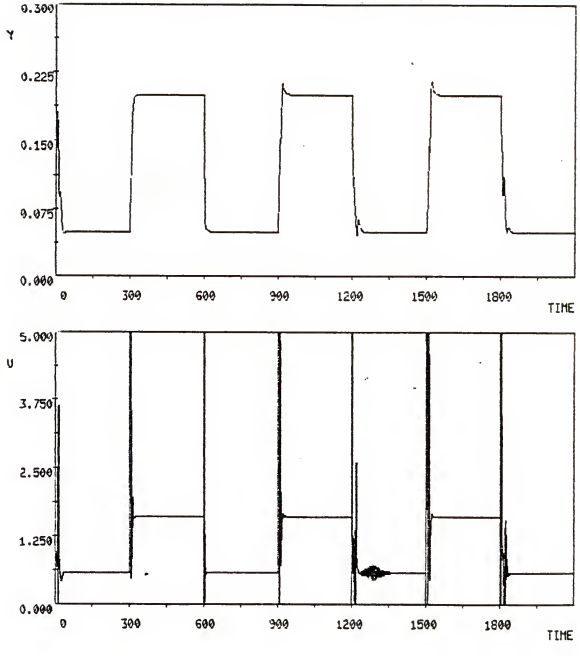


Figure 6-3. FAMDC with f=r=0.7 for set-point changes.

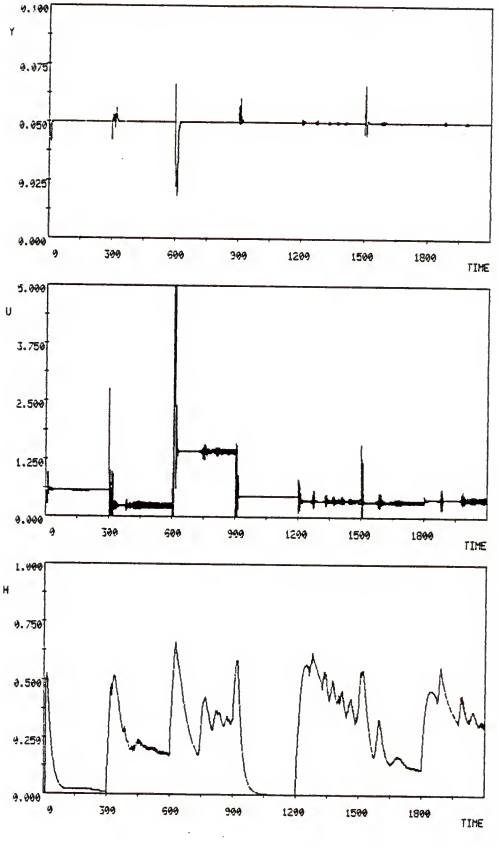


Figure 6-4. CAMDC with f(t)=r(t) for set-point changes.

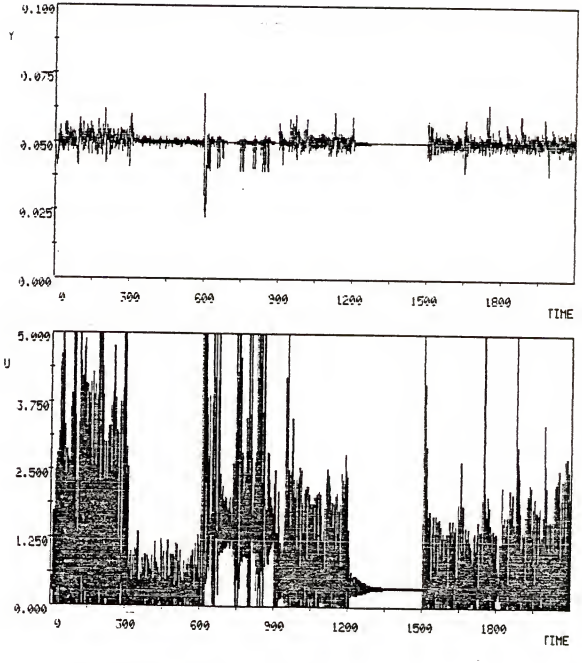


Figure 6-5. FAMDC with f=r=0 for load changes.

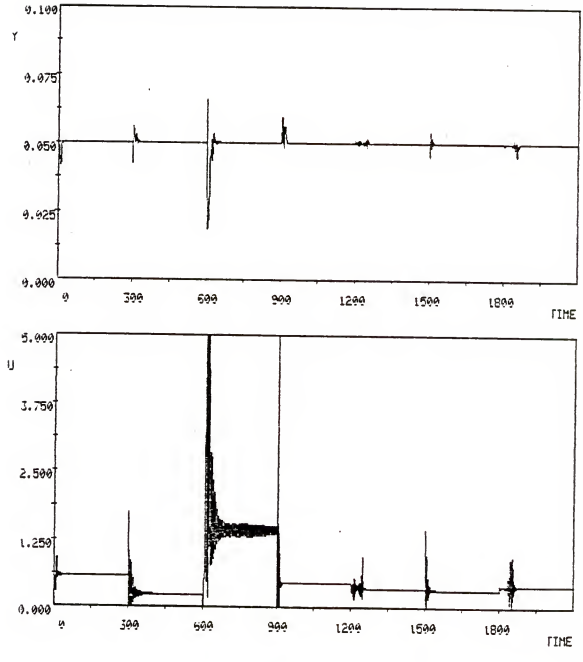


Figure 6-6. FAMDC with f=r=0.5 for load changes.

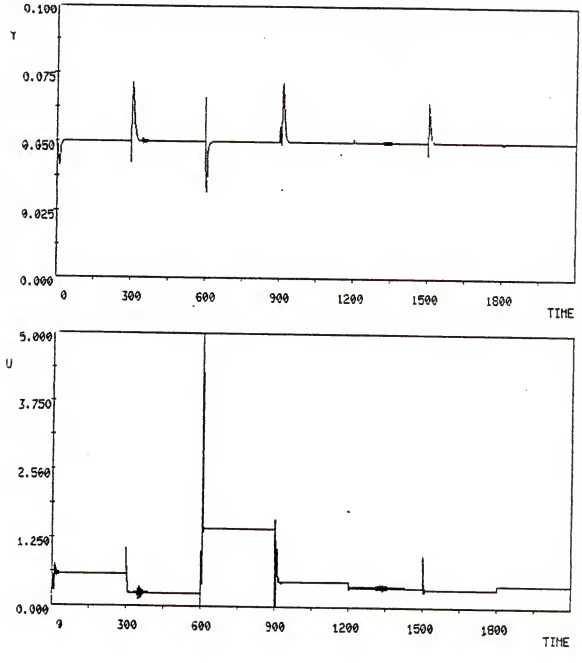


Figure 6-7. FAMDC with f=r=0.7 for load changes.

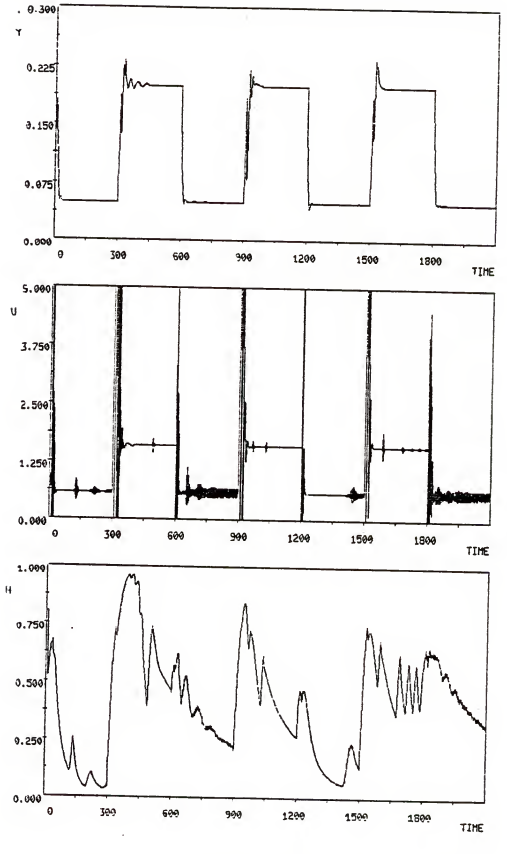


Figure 6-8. CAMDC with f(t)=r(t) for load changes.

CHAPTER VII
ADAPTIVE CONTROL FOR WASTEWATER TREATMENT PROCESSES

Introduction

Process control is unanimously accepted today as an effective way of improving the performance and reducing the operating costs of wastewater treatment processes. It is traditional design practice to increase the size of the process in order to accommodate large load variations. Recent improvements in sensor reliability and inexpensive computer systems make process control an attractive alternative to overdesign and a powerful method of operating the treatment process so that they meet the environmental standards at a minimum cost.

Since the inputs of a treatment process are usually varying with period of one day, the plants are rarely operating at steady-state conditions. For this reason, there has been a strong interest in the development of dynamic models for those processes. The importance of dynamic modeling and process control for the activated sludge process is effectively illustrated in the pioneering work of Andrews (1974) and Busby and Andrews (1975). Most of these models are nonlinear, time varying, of large dimensionality and involve a large number of parameters to be fitted. Simpler models have been also developed which adequately describe the dynamic performance of the activated sludge process. Such examples are transfer function models e.g. Lech et al. (1978a,b) and time series models e.g. Berthouex et al. (1978), and Debelak and Sims (1981).

The linear models are more appropriate for controller design whereas the complicated nonlinear ones are invaluable tools for process simulation and controller testing.

In a survey paper, Olsson (1977) discussed the problems of traditional treatment processes, the objective for their operation and the state-of-the-art control. The main control objectives for the activated sludge process can be summarized as: a) improve the quality and minimize the quality variation of the effluent stream, b) minimize the influence of feed diurnal variations on the physiological state of the biological matter, and c) reduce operating costs.

Standard control strategies today include: solids retention time (SRT) control, food to mass ratio (F/M) control, and sludge blanket level (SBL) control. Most of these strategies aim at improving the settleability of the biological floc and at keeping a constant amount of activated sludge in the system despite any load variations. Examples of such applications of process control are reported in Grinker and Meagher (1984) where the treatment efficiency was improved by more than 50% and in Wells (1978) where filamentous bulking was avoided and power expenses for dissolved oxygen control were lowered by 18%.

One of the most important variables in the activated sludge process is the dissolved oxygen (DO) concentration. DO control has drawn much attention in the recent years, e.g. Wells (1978), Olsson et al. (1985), Kay et al. (1985), and Fujimotto et al. (1983). Regulation of DO concentration at the proper level suppresses the development of filamentous organisms and their deleterious effects in the operation of the final settler. In addition, it minimizes power costs. The widespread application of DO control today has been facilitated by the development of reliable DO measurement probes.

The past unavailability of reliable sensors for measuring other process variables, e.g. the concentration of the active biomass or the BOD in the influent and in the effluent streams, hindered the development of effective control schemes for the effluent quality. As a result most of the schemes in practice today for such control use implicit measurements of the biological activity, or of the incoming load. For example Olsson and Andrews (1978) use the DO profile for estimating the organic loading, while Holmberg (1982) employs a state estimator based on DO and aeration rate observations.

Long term objectives, such as solids inventory maintenance, are easily achieved through the use of implicit measurements as described above. The more challenging objectives of minimizing both the pollutional load of the effluent stream and the variation of the effluent quality require more sophisticated measurement and control equipment. With the advent of more reliable sensors for the measurement of suspended solids and effluent quality, e.g. COD, TOC, or TOD, control schemes can now be developed which compensate for diurnal variations in a straightforward and more effect manner. Mixed liquor suspended solids (MLSS) measurement devices are currently in use in many plants (Flanagan (1978), Wells (1978)) with very successful results. On-line TOC and TOD analyzers have been recently introduced and successfully tested in practice (e.g. Stephenson et al. (1981), Tiemon and Wagner (1981)).

The estimation or measurement of the effluent quality permits direct assessment of the activated sludge process performance, in the case of large diurnal variations. Optimal control has been proposed by Cook and Marsili-Libelli (1981), and optimal periodic control by Sincic and Bailey (1978), and Yeung et al. (1980). If there is adequate sludge storage,

these methods result in excellent treatment performance. Their major disadvantage is that they require the use of complicated nonlinear dynamic models with a large number of parameters which are very difficult to identify either off-line or on-line.

In this work simple linear and bilinear dynamic models will be used for the optimal control of the activated sludge process. The advantage of linear models lies in their simplicity, ease of identification and flexibility in controller design. Since the process is characterized by time varying gains, time constants and parameters, the linear model is continuously identified on-line and adaptive control methods are employed for controller design.

Sincic and Bailey (1978) have determined that for physically realistic bounds on the control variable, the optimal control policy with respect to the effluent quality and variability is essentially bang-bang.

For these reasons, an optimal adaptive bang-bang controller is developed in this chapter which minimizes the performance measure that was introduced by Sincic and Bailey (1978). At the same time adaptive DO control is also implemented and the effectiveness of the proposed control strategies is illustrated by simulations with a simple process model.

Process Description - Model for Simulation

The activated sludge process is schematically represented in Figure 7-1. In the aeration basin organic matter from the influent stream comes into contact with aerobic biological matter (activated sludge) and is oxidized to cell mass, carbon dioxide and water. The aerobic conditions

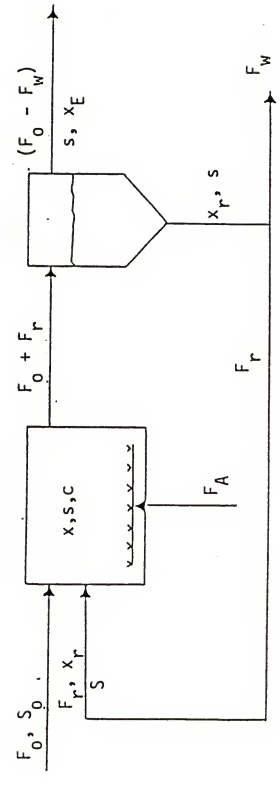


Figure 7-1. The activated sludge process.

are maintained by continuous aeration of the basin. The activated sludge is separated from the liquid phase in the final sedimentation tank and is partly recycled to the aeration basin in concentrated form. A portion of the sludge is wasted from the bottom of the sedimentation tank.

Aeration Basin

There are two major modeling approaches for the activated sludge process. The simpler one considers two aggregated state variables: the concentration of the growth limiting organic substrate (s) and the concentration of the viable or active biomass (x), e.g. Andrews (1971), Sincic and Bailey (1978). A better representation of the dynamic behavior of the process is obtained by structuring the substrate and the biomass into several components, e.g. Busby and Andrews (1975), Nelson and Lawrence (1980), and Curds (1971, 1973). Although these models are closer to reality than the aggregated ones, they usually result in a large number of differential equations which are difficult to verify experimentally.

In this work, for the sake of simplicity, the algorithm will be tested with an aggregated variables model simulating the process. The aeration basin is modeled as an ideal continuous flow stirred tank reactor. In addition, the following further simplifying assumption are made: i) the microorganism concentration in the influent stream is zero, ii) the substrate concentration in the return activated sludge stream is equal to the substrate concentration in the aeration basin. Under the assumptions the aeration basin dynamics are described by the following equations:

$$\frac{ds}{dt'} = \frac{F_o s_o}{V} - \frac{F_o}{V} s - \frac{1}{Y} \mu(s,c)x \tag{7-1}$$

$$\frac{dx}{dt'} = \frac{F_r x_r}{V} - \frac{(F_o + F_r)x}{V} + \mu(s,c)\, x - k_d x \tag{7-2}$$

$$\mu(s,c) = \frac{\mu_{max}}{k_s + s} \frac{c}{k_c + c} \tag{7-2}$$

where $\mu(s,c)$ is the biomass specific growth rate, and the notation for the rest of the variables is given in Table 7-2 at the end of the chapter. Here the biomass specific growth rate follows Monod kinetics with double substrate limitation by both the substrate concentration s and the dissolved oxygen concentration c.

These two differential equations for the aeration basin describe the process sufficiently well for the needs of this work.

Sedimentation Tank

The sedimentation tank accomplices two important tasks. As a "clarifier" it separates the activated sludge from the treated water. As a "thickener" it concentrates the biological solids which are recycled and serves as a sludge storage reservoir. The performance of the activated sludge process crucially depends on the efficiency of both functions. If the sludge has poor settling properties or the clarifier cannot accommodate incoming loads, the solids are carried away with the supernatant water and increase the BOD in the effluent stream. If, on the other hand, the recycled sludge is not concentrated enough, then it may not be possible to recycle the amount of solids required to compensate for large incoming loads.

A sludge mass balance around the final sedimentation tank gives

$$\frac{dM_c}{dt'} = (F_o + F_r)\, x - F_r x_r - (F_o - F_w)\, x_E - F_w x_r \qquad (7\text{-}4)$$

where M_c is the total mass of solids in the tank.

In order to completely describe the dynamic behavior of the thickener we need to know the concentration profile in the thickening volume and the effluent solids concentration. Partial differential equations and the "limiting flux theory" have been employed to describe dynamically the thickening and clarifying functions, e.g. Tracy and Keinath (1973), George and Keinath (1978). Empirical steady-state equations for the effluent solids concentration have been used by (e.g. Pflanz (1969), Ghobrial (1978)). The validity of both approaches is limited and in general the dynamic phenomena associated with the sedimentation tank are poorly understood.

In this work the dynamic behavior of the clarifier is treated in a qualitative way. Two extremes in clarifier performance are considered with the actual process lying somewhere in between. If the sedimentation tank is small enough, its dynamics are much faster than the dynamics of the aeration basin and the pseudosteady-state hypothesis is approximately valid. In this case the return activated sludge concentration is given by

$$x_r = \frac{(F_o + F_r)\, x - (F_o - F_w)\, x_E}{F_r + F_w} \qquad (7\text{-}5)$$

Pflanz (1969) gives the following empirical relation for the effluent solids concentration x_E

$$x_E = K_p(F_o + F_r)\, x \tag{7-6}$$

We henceforth refer to this as the "fast" settler case.

At the other extreme, we consider a large sedimentation tank which provides for sludge storage volume and is practically insensitive to any changes in the influent or recycle flowrates. In this case the return sludge concentration is characterized by very slow dynamics and for all practical purposes can be assumed constant, i.e.

$$x_r(t) = x_r \tag{7-7}$$

In the sequel this will be referred to as the "slow" settler case.

It has been recently realized that successful application of control policies aiming at the improvement of performance with respect to diurnal variations requires a large storage volume for concentrated sludge. This volume can be incorporated in the final sedimentation tank by increasing its size (Flanagan (1978)) or it can be provided by an external sludge reservoir. Such modifications have been discussed by Nogita and Hiraoka (1981), Nogita et al. (1981), and Lohmann and Schlegel (1981).

Dissolved Oxygen Dynamics

The oxygen mass transfer and utilization are the best understood phenomena in the activated sludge process. Oxygen is used by the

microorganisms for respiration during biomass synthesis and endogenous metabolism. The consumed oxygen is replaced through strong aeration of the reacting volume. The DO mass balance is given by the following differential equation:

$$\frac{dc}{dt'} = -\frac{F_o + F_r}{V} c + K_{La}(c^s - c) - OUR \tag{7-8}$$

$$OUR = p_1\mu(s,c)\, x + p_2 x \tag{7-9}$$

In developing this equation it is assumed that the influent and recycle streams carry a negligible amount of dissolved oxygen. The parameter K_{La} represents the oxygen mass transfer coefficient and is assumed to be a linear function of the air flow rate F_a

$$K_{La} = p_3 + p_4 F_a \tag{7-10}$$

The last term in (7-8) represents the oxygen utilization rate for both biosynthesis and biodegradation. The OUR is one of the most important variables in many control approaches since it is closely related to the biological activity inside the aeration basin.

Control Objectives and Algorithms

The adaptive control algorithms developed in this chapter aim at two control objectives. First, the quality of the effluent is of paramount importance. In particular, it is desired to minimize the effects of the periodic load variations on the quality of the effluent stream. Second,

since the DO concentration in the aeration basin can greatly influence the sludge settling characteristics (e.g. Lau et al., 1984; Fujimoto and Arai, 1981), it is essential to maintain DO within a certain range. Use of feedback control to achieve the latter objective can in addition result in substantial energy savings (e.g. Speirs and Stephenson, 1981).

The effluent quality is best influenced by manipulation of the return activated sludge (RAS), and the DO concentration can be controlled through the aeration rate F_a. Since the DO control loop has response time of the order of minutes, whereas the effluent quality loop is much slower with response time of the order of hours, the two control loops can be considered independently.

Effluent Quality Control

A performance measure is explicitly formulated and a linear dynamic model is utilized to design a feedback/feedforward adaptive control law which minimizes the performance measure. The approach followed here was motivated by the original work of Sincic and Bailey (1978) and Yeung et al. (1980).

The activated sludge process performance with respect to the effluent quality, may be represented by the following performance measure:

$$J(t') = \frac{q}{T_p} \int_{t'}^{t'+T_p} (F_o(\tau)-F_w)y(\tau)d\tau + \frac{1}{T_p} \int_{t'}^{t'+T_p} (y(\tau)-y_{ave})^2 d\tau \quad (7\text{-}11)$$

$$y(t') = p_5 s(t') + p_6 x_E(t') \quad (7\text{-}12)$$

where

- $y(t')$ is a measurement of the effluent quality, e.g. TOC or TOD, accounting for both filterable substrate (s) and solids (x_E)
- T_p is the period of the feed variations, typically one day
- and $q \geq 0$ is a weighing coefficient.

The first term in the performance measure represents the daily amount of substrate discharged with the effluent stream and the second term the variation of the effluent quality from the average value y_{ave}. As shown later this can be chosen as either the mass average or the average y. To ensure that y_{ave} is within acceptable limits, the following performance constraint is imposed:

$$y_{ave} \leq y_{max} \tag{7-13}$$

In addition there are constraints imposed by the process, namely the RAS rate must be restricted between two values, $F_{r,min}$ to avoid washout of the aeration basin and $F_{r,max}$ to avoid depletion of the clarifier blanket. Sincic and Bailey (1978) showed that for realistic RAS bounds the optimal controller which minimizes (7-11) is bang-bang (or very nearly so). Using this fact, the optimization task is made easier by restricting the search to the optimal bang-bang controller.

For adaptive control the parameters of a discrete time input-output model (usually linear) are continually updated on line. The process output is the effluent quality measurement as given by (7-12). Process inputs are the control variable Fr, the influent flow rate F_o and, if measured or estimated, the influent load $F_o s_o$.

A second order time series input-output model is employed to desribe the dynamic behavior of the process

$$y(t+1) = a_o y(t) + a_1 y(t-1) + b_o u(t) + b_1 u(t-1) \qquad (7\text{-}14)$$

$$+ c_o d(t) + c_1 d(t-1) + e_o l(t) + e_1 l(t-1) + r$$

where

- t represents time in number of sampling periods
- y(t+1) is the process output
- u(t) is the control variable $F_r(t)$
- d(t) is the influent flowrate $F_o(t)$
- l(t) is the influent load $F_o s_o(t)$
- a_i, b_i, c_i, e_i (i=0,1) and r are the on-line identified model parameters.

If the influent load is not measured, it can be excluded form the model by taking $e_0 = e_1 = 0$.

The unknown parameters of model (7-14) are estimated on-line via a recursive least squares estimator with a variable forgetting factor (RLSVFF) using input and output information (Fortescue et al., 1981).

The adaptive optimal bang-bang control problem is then formulated as follows:

Minimize

$$J_D(t) = \frac{q}{N} \sum_{i=1}^{N} (\hat{d}(t+i)-F_w)\hat{y}(t+i) + \frac{1}{N} \sum_{i=1}^{N} (\hat{y}(t+i)-\hat{y}_{ave})^2 \qquad (7\text{-}15)$$

(which results from the discretization of performance measure (7-11))

subject to

$$\text{(i)}\quad \hat{y}(t+i) = \hat{a}_o(t)\hat{y}(t+i-1) + \hat{a}_1(t)\hat{y}(t+i-2) \qquad (7\text{-}16)$$

$$+ \hat{b}_o(t)u(t+i-1) + \hat{b}_1(t)u(t+i-2)$$

$$+ \hat{c}_o(t)\hat{d}(t+i-1) + \hat{c}_1(t)\hat{d}(t+i-2) + \hat{r}(t)$$

$$+ (\hat{e}_o(t)\hat{l}(t+i-1) + \hat{e}_1(t)\hat{l}(t+i-2)) \qquad \text{for } i \geq 1$$

$$\hat{y}(t+i) = y(t+i) \qquad \text{for } i = -1,0$$

$$\text{(ii)}\quad \hat{y}_{ave} \leq y_{max} \qquad (7\text{-}17)$$

over all possible control vectors

$$\underline{v}_i = [\ldots, u_i(j), \ldots]^T \qquad \begin{matrix} i = 1, \ldots, M \\ j = t, \ldots, t+N-1 \end{matrix} \qquad (7\text{-}18)$$

where

- $u_i(j)$ is either u_{min} or u_{max}
- M is the number of all possible combinations of u_{min} and u_{max} to form the control vector v_i, i.e. $M = 2^N$.
- $\hat{y}(t+i)$ denotes the prediction of the output at time t+i using the current parameter estimates $\hat{r}(t)$, $\hat{a}_j(t)$, $\hat{b}_j(T)$, ..., (j=0,1).
- $N = T_P/T$ with T being the sampling period

$$\hat{y}_{ave} = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N} \hat{y}(t+i) & \text{(a)} \\ \text{or} & \\ \dfrac{\sum_{i=1}^{N} (\hat{d}(t+i)-F_w)\hat{y}(t+i)}{\sum_{i=1}^{N} (\hat{d}(t+i)-F_w)} & \text{(b)} \end{cases} \qquad (7\text{-}19)$$

The first choice accounts for the variability in the effluent concentration, whereas the second accounts for the variability of the mass averaged effluent concentration.

Future feed variables ($\hat{d}$ and $\hat{l}$) may be predicted from the process operation history. Past data can be analyzed with the aid of statistical methods to obtain daily average influent profiles or to forecast future values. Such analyses have been reported by Berthouex et al. (1978) and Debelak and Sims (1981). The simplest but probably not the best method is to take the anticipated feed variables equal to those of the previous day at the same time.

The above problem is solved combinatorially. The performance measure $J_D(t)$ is computed for each of the control vectors v_i. The optimal control vector v_{opt} is the one correponding to the minimum value of $J_D(t)$ which does not violate the constraint on the effluent average (7-17).

Because the estimator parameters are updated at every sampling instant, only the first element of $v_{opt}(t)$, $u_{opt}(t)$ is implemented. At the next sampling interval $t + 1$ the minimization is performed again using the updated model parameters and the current inputs and output.

The sampling period T is determined by the process dynamics, the measurement devices and the feed period. The fact that the activated sludge process has slow dynamics for the RAS - effluent quality loop suggests that a large sampling period is appropriate (typically in the range 1-4 hours). Current measurement devices for TOC or TOD can easily accommodate such sampling requirements.

Given the large sampling period, the required computations are a rather simple task for contemporary control computers. For example, if N = 8 (T = 3 hours) $J_D(t)$ is calculated 256 times. This task can be accomplished in seconds by most microcomputers available today.

One final remark is in order. If the control stays constant for many sampling periods, the parameter estimator may run into difficulties. To avoid this, a small amount of excitation is added to the calculated $u_{opt}(t)$.

Dissolved Oxygen Control

The control objective for the DO loop is to maintain the dissolved oxygen concentration c(t) at the desired value c_d. Adaptive control is the natural framework for this loop since many system parameters (e.g. OUR, K_{La}) are experiencing relatively slow variations.

Equation (7-8) suggests that a bilinear model can be used to describe the DO dynamics

$$z(k+1) = m_1 z(k) + m_2 w(k) + m_3 z(k)w(k) + m_4[d(k) + u(k)]z(k) + m_5 \quad (7\text{-}20)$$

where

- k represents time in number of sampling periods T_k for the DO loop (which is typically 3-10 minutes)
- z(k) is the DO measurement
- w(k) is the aeration rate F_a
- m_i i = 1, 4 are model parameters
- d(k), u(k) are the samples of F_o, F_r at time k.

As in the case of effluent quality control the parameters m_i are estimated with the RLSVFF estimator.

A robust adaptive controller is desired for the fast DO loop. We shall employ the cautious self tuning controller (CSTC) developed in Chapter III, which is based on the following performance measure:

$$J_{ox} = [\hat{z}(k+1) - c_d]^2 + Q(k)[w(k) - w_c]^2 \qquad (7\text{-}21)$$

where

- $\hat{z}(k+1)$ is the output prediction at time k

$$\hat{z}(k+1) = \hat{m}_1(k)z(k) + \hat{m}_2(k)w(k) + \hat{m}_3(k)z(k)w(k) \qquad (7\text{-}22)$$

$$+ \hat{m}_4(k)[d(k)+u(k)]z(k) + \hat{m}_5(k)$$

- w_c is a safe control, as for example a constant aeration rate, high enough to guarantee sufficiently high DO concentration even at large loads.

Employing similar analysis as in Chapter III, it can be proved that an appropriate weight which provides robust control without unnecessarily slowing down the process is

$$Q(k) = \frac{h(k)[\hat{m}_2(k) + \hat{m}_3(k)z(k)]^2}{1 - h(k)} \tag{7-23}$$

where

$$h(k) = \frac{\nu(k)}{\delta|[\hat{m}_2(k) + \hat{m}_3(k)z(k)]w_c| + \nu(k)} \tag{7-24}$$

$$\nu(k) = \lambda\nu(k-1) + (1-\lambda)|\varepsilon(k)| \; ; \qquad 0 \leq \lambda < 1 \tag{7-25}$$

($\nu(k)$ is a filtered estimation error $\varepsilon(k)$)

$$\varepsilon(k) = z(k) - \hat{z}(k) \tag{7-26}$$

The parameter δ is the maximum allowable relative error in the control law and is chosen in the range of 0.05-0.5.

Minimization of (7-21) with respect to w(k) results in the following control law:

$$w(k) = h(k)w_c + [1-h(k)]w_{MV}(k) \tag{7-27}$$

where w_{MV} is the dead-beat controller obtained by minimizing J_{ox} with $Q(k) = 0$. This control law sets

$$\hat{z}(k+1) = c_d \tag{7-28}$$

i.e. drives the output to the set point at the shortest possible time.

Simultation Results

To simulate the process, the model of (7-1)-(7-3), (7-8)-(7-10) was used for the aeration basin. Both the "fast" (7-5), (7-6) and the "slow" (7-7) extremes for the sedimentation tank were considered. The model parameters used in the simulations are presented in Table 7.1

The inlet flowrate F_o and substrate concentration s_o were assumed to be approximately periodic

$$F_o(t) = \bar{F}_o(1+(h_F + N_1(t))\sin(2\pi t/T_p)) \tag{7-29}$$

$$s_o(t) = \bar{s}_o(1 + (h_s + N_2(t))\sin(2\pi t/T_p)) \tag{7-30}$$

where $\bar{F}_o$, $\bar{s}_o$ are the average values of F_o, s_o and the coefficients h_F, h_s determine the amplitude fo the oscillation. White noise $N_1(t)$, $N_2(t)$ was added to simulate day to day variations. The F_o, s_o as given by (7-29), (7-30) are shown in Figure 7-2.

Equation (7-19a) was used for y_{ave} in the optimization algorithm, and the maximum allowable average y_{max} was set at 15 mg/lt. The recycle flow rate is bounded between the u_{min} and u_{max} given in Table 7-1. The lower and upper bounds $F_{a,min}$, $F_{a,max}$ for the aeration flow rate are also given in the same table.

Testing of the effluent quality control algorithm was first undertaken assuming a constant DO concentration of 2 mg/lt in the aeration basin. In Figure 7-3 the performance of the optimal bang-bang controller is shown for the "slow" settler case and q = 0. In this case only the effluent variability is minimized subject to the constraint (7-17). The average effluent concentration is found to be 12.5 mg/lt, whereas the average value of the performance measure is calculated to be $J_{d,ave}$ = 8.2 mg/lt.

In order to study the effect of the parameter q in the performance measure (7-15), the same run was repeated taking q = 0.01. As seen from Figure 7.4 the average y is somewhat lower (9.3 mg/lt) at the expense of variability (15.4 mg/lt compared to 8.2 mg/lt for the previous run).

To compare the effectiveness of the proposed control algorithm with that of standard wastewater treatment control practice, the performance of ratio control was evaluated. For comparison purposes, the ratio was chosen so as to achieve the same y_{ave} as in the case of Figure 7-4. The temporal variation of the effluent quality measurement is shown in Figure 7-5. In this case the average variability (27.11 mg/lt) is significantly higher than in the adaptive bang-bang case.

However, as seen in Figures 7-6 and 7-7, in the case of the fast settler model the improvement with adaptive optimal bang-bang control is only marginal (variabilitiy 51.8 mg/lt versus variability 61.0 mg/lt for ratio control with the same y_{ave}). With realistic settler dynamics an intermediate performance is expected. It is obvious that the larger the settler size is, the more effective can control action be in compensating for diurnal variations.

In order to test the performance of the two control loops combined, the "slow" settler case was repeated with CSTC DO control. As seen from Figure 7-8 the DO loop operates successfully and the effluent quality is comparable to that of Figure 7-4.

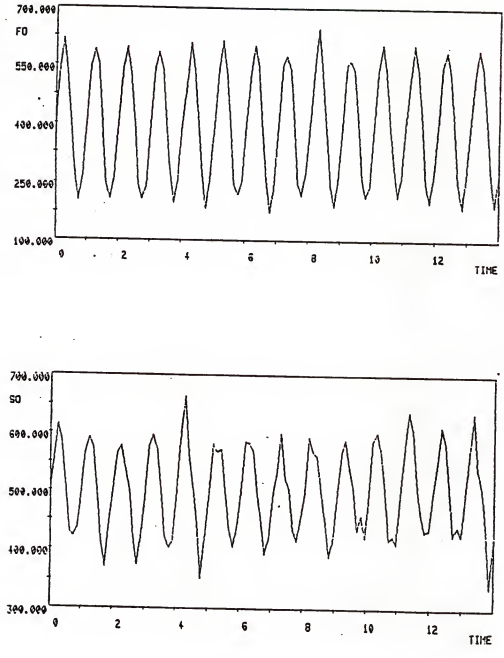


Figure 7-2. Influent flowrate F_0 and substrate concentration s_0 versus time in days.

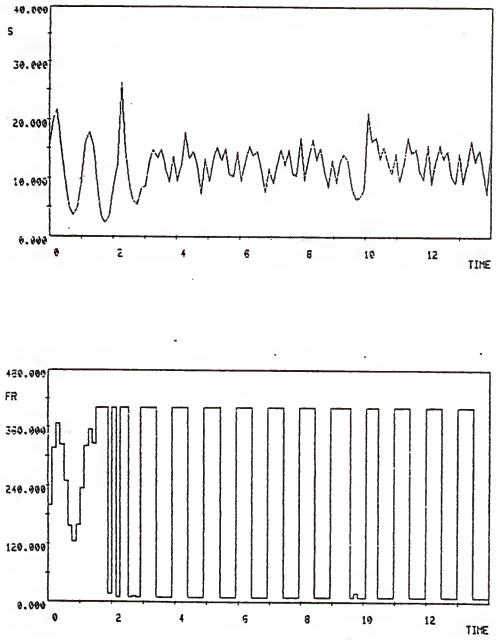
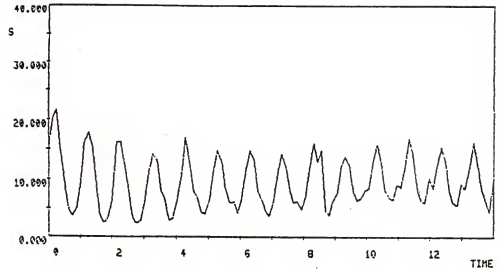


Figure 7-3. Effluent substrate concentration s and recycle flowrate F_r for "slow" settler dynamics with adaptive optimal bang-bang control (q=0). Perfect DO control is assumed.



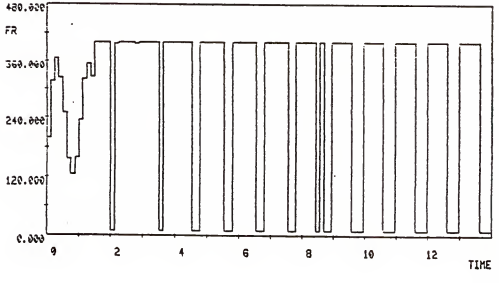


Figure 7-4. Effluent substrate concentration s and recycle flowrate F_r for "slow" settler dynamics with adaptive optimal bang-bang control (q=0.01). Perfect DO control is assumed.

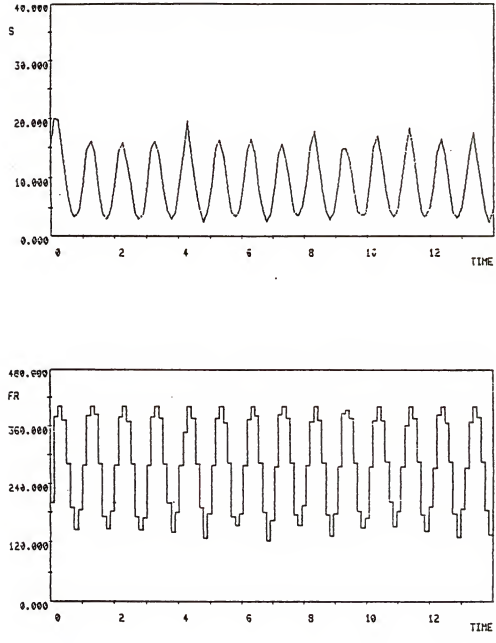


Figure 7-5. Effluent substrate concentration s and recycle flowrate F_r for "slow" settler dynamics with ratio control (r=0.7). Perfect DO control is assumed.

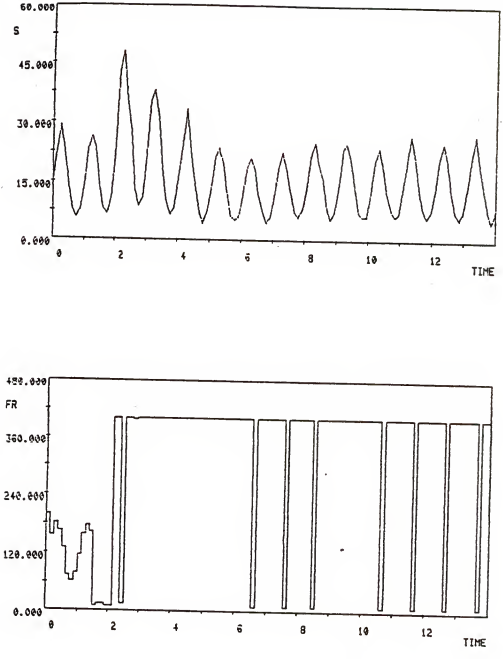
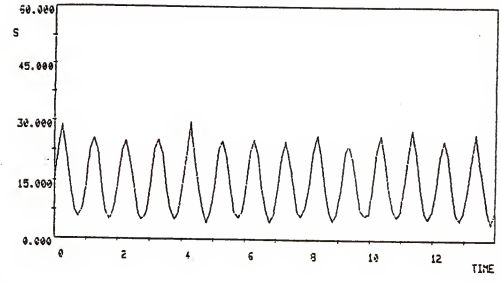


Figure 7-6. Effluent substrate concentration s and recycle flowrate F_r for "fast" settler dynamics with adaptive optimal bang-bang control (q=0.01). Perfect DO control is assumed.



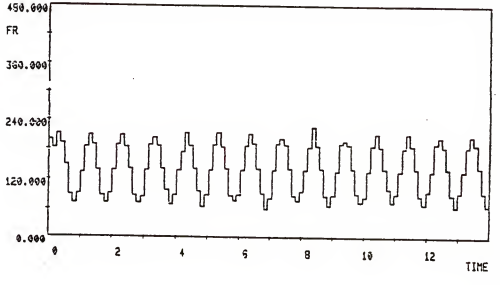
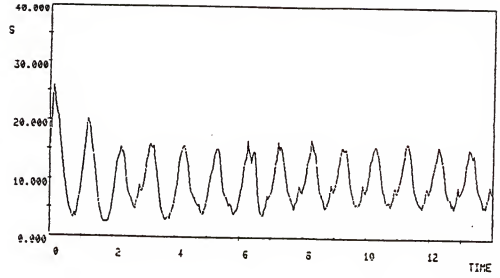


Figure 7-7. Effluent substrate concentration s and recycle flowrate F_r for "fast" settler dynamics with ratio control (r=0.35). Perfect DO control is assumed.



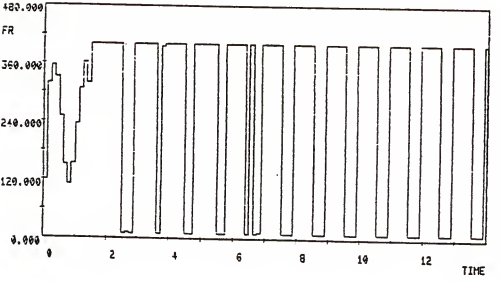
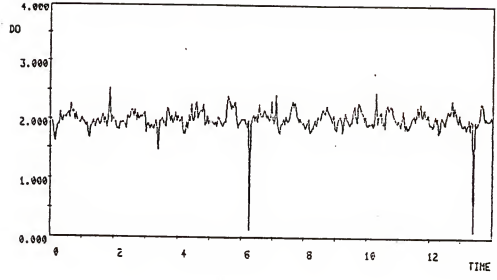


Figure 7-8. Effluent substrate concentration s and recycle flowrate F_r for "slow" settler dynamics with cautious self-tuning control of DO and adaptive optimal bang-bang control for effluent quality (q=0.01).



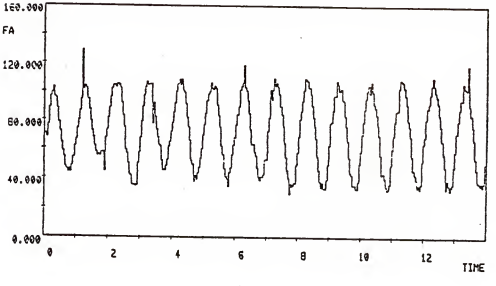


Figure 7-9. DO concentration and aeration flowrate F_a for "slow" settler dynamics with cautious self-tuning control of DO and adaptive optimal bang-bang control for effluent quality (q=0.01).

Table 7-1

Parameter Values

Aeration Basin

c^s = 9 mg/lt

F_o = 0.111 m^3/sec

F_w = 2.444×10^{-3} m^3/sec

h_F = 0.5

h_s = 0.2

K_c = 5×10^{-2} mg/lt

K_d = 1.389×10^{-6} sec^{-1}

K_s = 140 mg/lt

μ_{max} = 5.7×10^{-5} sec^{-1}

N = 8

$N_1(t)$ = white noise sequence with variance 1

$N_2(t)$ = white noise sequence with variance 1

P_1 = 1

P_2 = 0.9

P_3 = 0

P_4 = 0.1

P_5 = 1

P_6 = 0

s_o = 5 mg/lt

T = 3 hr (108000 sec)

T_k = 0.1 hr (360 sec)

T_p = 8.64×10^4 sec (1 day)

u_{max} = 0.111 m^3/sec

Table 7-1 - continued

u_{min} = 2.778×10^{-3} m^3/sec

V = 2500 m^3

Y = 0.4

y_{max} = 15 mg/lt

Sedimentation Tank

A = 300 m^2

K_p = 4.32 sec/m

x_r = 6500 mg/lt

CSTC Parameters

c_d = 2 mg/lt

$F_{a,max}$ = 4.167×10^{-2} m^3/sec

$F_{a,min}$ = 0 m^3/sec

w_c = 2.778×10^{-2} m^3/sec

δ = 0.2

λ = 0.95

Table 7-2

Notation

A	: sedimentation tank cross section area
a_i	: model (7-14) parameters
b_i	: model (7-14) parameters
$c(t)$	: dissolved oxygen concentration
c^s	: saturated dissolved oxygen concentration
c_i	: model (7-14) parameters
c_d	: dissolved oxygen concentration set point
$d(t)$	: influent flow rate $F_o(t)$, sampled at time t
e_i	: identified model parameters
F_a	: aeration flow rate
F_o	: influent flow rate
F_r	: recycle flow rate
F_w	: waste stream flow rate
h	: CSTC control law weight
h_F	: parameter in equation (7-28)
h_s	: parameter in equation (7-29)
J	: performance measure
k	: time in number of sampling periods in the DO control loop
K_c	: saturation constant for oxygen Monod expression
K_d	: microorganism decay coefficent
K_{La}	: overall oxygen mass transfer coefficient
K_p	: portionality coefficient in equation (7-6)
K_s	: saturation constant for substrate Monod expression

Table 7-2 - continued

$l(t)$: $F_o s_o$ the influent waste load

M_c : solids mass in the sedimentation tank

m_i : model (7-20) parameters

N : number of sampling intervals in influent period

$N_i(T)$: white noise sequence

P_i : constant parameters in equations (7-9), (7-10), (7-12)

q : performance index weight in equations (7-11), (7-15)

$Q(k)$: performance index weight in equation (7-21)

r : model (7-14) parameter

s : substrate concentration in the aeration basin

s_o : influent substrate concentration

t : time in number of sampling intervals for the effluent-RAS loop

t' : continuous time

T : sampling period for the effluent quality loop

T_k : sampling period of the DO loop

T_p : period of influent variations

$u(t)$: return activated sludge (RAS) flow rate

u_{min} : lower bound of RAS

u_{max} : upper bound of RAS

$u_{opt}(t)$: optimal control law at time t

V : aeration basin volume

v_i : control vector

v_{opt} : optimal control vector

w_c : "safe" aeration rate

Table 7-2 - continued

$w(k)$: aeration rate

x : active biomass concentration in the aeration basin

x_E : active biomass in the effluent stream

x_r : active biomass concentration in the recycled stream

y_{ave} : average effluent quality over a period

y_{max} : maximum allowable y

$y(t)$: measurement of the effluent quality

Y : growth yield factor

$z(k)$: sampled dissolved oxygen concentration

Greek Letters

μ_{max} : maximum growth rate

ν : filtered estimation error

δ : CSTC design parameter

λ : CSTC filter parameter

ε : estimation error

Subscripts

o : denotes influent variable

r : denotes recycle variables

E : denotes effluent stream variables

ave : denotes average values

max : denotes maximum values

min : denotes minimum values

CHAPTER VIII
SUMMARY AND CONCLUSIONS

Adaptive controllers based on linear models face several problems when applied to highly nonlinear systems, as for example, large sustained oscillations, failure in set-point tracking, or even instability. Three control methods have been developed here for the adaptive control of processes which considerably deviate from the linear behavior.

The first controller is a modification of well known self-tuning control algorithms and is called the "stabilizing" STC. This is a bounded "standard" STC if the magnitude of both the estimation and tracking error are small, and switches to a robust conservatively tuned internal model controller if one of the magnitudes become large. Under the modified algorithm, simulations with a nonlinear system show that the closed-loop system remains stable for much larger set-point and load changes, than it does under "standard" self-tuning control.

The second controller called cautious self-tuning controller links a conservative robust control law and the minimum variance control law, which is a controller resulting to fast closed-loop dynamics. The CSTC adjusts the weighing between its two parts, giving conservative control when the parameter estimates are poor, but speeding up system response as the estimates improve. Although it requires a low amount of modeling information it performs well for both servo-tracking and regulation, even in the presence of strong nonlinearities. The algorithm resolves effectively the weight selection problem often encountered in one-step ahead optimal quadratic control applications. The computational

requirements are minimal, only slightly higher than those of the minimum variance STR. These features make CSTC an attractive alternative for the control of chemical plants, which are nonlinear and for which reliable dynamic models do not generally exist.

The above control methods can be applied only to minimum phase and open-loop stable systems. For nonminimum phase and/or open-loop unstable systems the model dependent controller is presented. MDC is a powerful design method which treats effectively instability, inverse response and time delays. It can handle independently set-point tracking and disturbance rejection assigning different closed-loop dynamics to each of them. This is achieved through two tuning parameters f and r that specify the location of the closed-loop poles. In case of model uncertainty the same parameters directly adjust the tradeoff between robustness and speed of response. Given uncertainty bounds as a function of frequency a robust MDC controller can be designed even if the system is open-loop unstable. If no uncertainty bounds are known and the system is open-loop stable then high enough f and r ensure closed-loop stability.

Two adaptive versions of the MDC law have been developed and they constitute the third method for the adaptive control of nonlinear processes. Stability and convergence of both algorithms have been established for linear time-invariant systems. The flexibility of the MDC design method in adjusting the tradeoff between robustness and performance is demonstrated through simulations on a typical chemical process. A method for adjusting automatically this tradeoff based on the plant/model mismatch has been presented and shown to be effective in controlling the nonlinear chemical process mentioned above.

Finally, an adaptive controller specifically designed for the activated sludge process for wastewater treatment has been developed. Controller design for the activated sludge process is complicated not only because the process is nonlinear but in addition because the flowrate and concentration of the influent waste are varying periodically with period of one day. The periodically of the inputs causes serious operational problems, the most serious being that the effluent quality may not meet the environmental standards. The adaptive controller presented here is of the bang-bang form and minimizes a process performance index which includes two different measures of the effluent quality: the average waste concentration at the effluent stream and its variance over a period. Numerical simulations have shown that the develcped adaptive bang-bang controller outperforms conventional control strategies, and that there is a clear tradeoff between the two measures of the effluent quality. Low average concentrations can be achieved at the expense of effluent variance.

For the activated sludge process dissolved oxygen control has been also successfully carried out using the CSTC extended to bilinear systems. Numerical simulations have verified that effluent quality control and DO control can be carried out simultaneously and independently.

APPENDIX A
INTERNAL MODEL CONTROL

Controller

Consider the z-domain representation of a linear single-input single-output (SISO) discrete system

$$Y(z) = G_0(z)\, U(z) + \sum_{i=1}^{r} G_{di}(z)\, D_i(z) \qquad \text{(A-1)}$$

where $Y(z)$, $U(z)$, $D_i(z)$ are the z-transforms of the output $y(t)$, the control variable $u(t)$ and the ith disturbance $d_i(t)$; $G_0(z)$ and $G_{di}(z)$ are the system transfer functions with respect to the control and the ith disturbance variable.

According to the IMC methodology the classical feedback structure of Figure A-1 is recast into the IMC structure of Figure A-2. The two structures are equivalent provided that

$$G_I = \frac{G_c}{1+G_c\, G} \qquad \text{(A-2)}$$

or

$$G_c = \frac{G_I}{1-G\, G_I} \qquad \text{(A-3)}$$

where G is the nominal system model.

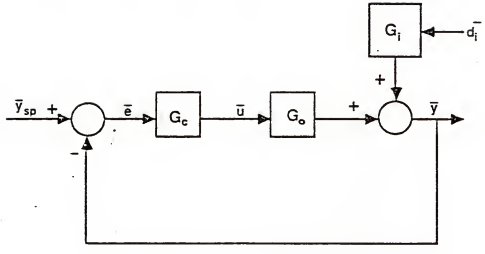


Figure A-1. The classical feedback structure.

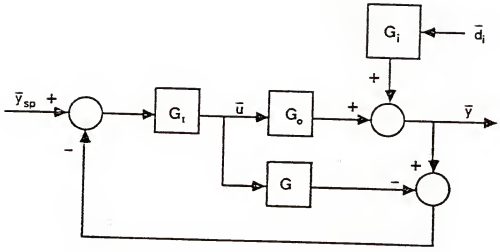


Figure A-2. The internal model control (IMC) structure.

In IMC the controller design is carried out through G_I and the implemented controller transfer function is then obtained from (A-3). The model G is factored as

$$G(z) = G_-(z)\, G_+(z)$$

where $G_+(z)$ contains all the non-invertible elements of G (e.g. time delays and zeroes outside the unit circle). $G_I(z)$ is then chosen as an approximate model inverse

$$G_I(z) = G_-^{-1}(z)\, F(z) \tag{A-4}$$

where the "filter" F(z) introduces extra dynamics as for example low pass filtering to increase robustness.

From (A-3) the controller transfer function is given then by

$$G_c(z) = \frac{F(z)}{G_-(z)\,[1-G_+(z)F(z)]} \tag{A-5}$$

If the model is perfect ($G = G_0$ the IMC closed-loop equation is

$$Y(z) = F_+(z)\, Y_{sp}(z) + [1-F_+(z)] \sum_{i=1}^{r} G_{di}(z)\, D_i(z) \tag{A-6}$$

where

$$F_+(z) = G_+(z)\, F(z)$$

and $Y_{sp}(z)$ is the set-point.

It is seen from (A-6) that the closed-loop poles for set-point changes involve only the filter poles. To eliminate offset, the filter steady-state gain should satisfy.

$$F_+(1) = G_+(1)\, F(1) = 1$$

Since (A-5) involves as zeroes the open-loop poles of (A-1), poles-zero cancellations occur in closed-loop. For this reason the IMC methodology is applicable only to open-loop stable systems.

Another drawback of IMC can be seen from (A-6). The disturbance term includes the filter poles in addition to the disturbance transfer function poles. If one of the latter poles is close to the unit circle the disturbance rejection will be slow.

Filter Design

Let F(z) be factored as

$$F(z) = F_1(z)\, F_2(z) \qquad \text{(A-7)}$$

and $F_1(z)$ is chosen to satisfy

$$|G_+(z)\, F_1(z)| = 1 \quad \text{for} \quad |z| = 1 \qquad \text{(A-8)}$$

with pole(s) inside the unit circle.

Note that if G(z) has no zeroes outside the unit circle, then

$$G_+(z) = z^{-k}$$

where k is the model time delay. In this case

$|G+(z)| = 1$ and we take $F_1 = 1$.

If G(z) involves zeroes outside the unit circle, then

$$G_+(z) = z^{-k} \prod_{i=1}^{m} (1-z_i^+ z^{-1})$$

where z_i^+ is a zero of G(z) outside the unit circle. Equation (A-8) is satisfied if

$$F_1(z) = \prod_{i=1}^{m} \frac{1}{-z_i^+ + z^{-1}} \qquad \text{(A-9)}$$

$G_+(z)\ F_1(z)$ form what is called a Blaschke product or an "allpass" filter (Zames and Francis, 1983).

The usual choise for $F_2(z)$ is

$$F_2(z) = \frac{1-f}{1-fz^{-1}} \qquad 0 \leq f < 1 \qquad \text{(A-10)}$$

Combining (A-9) and (A-10) the filter is

$$F(z) = \frac{1-f}{1-fz^{-1}} \prod_{i=1}^{m} \frac{1}{-z_i^+ + z^{-1}} \qquad \text{(A-11)}$$

Note that the above selection of F(z) results to

$$|F_+(z)| = |F_2(z)| \quad \text{for} \quad |z| = 1$$

and

$$F_+(1) = 1$$

Robustness of IMC

Assume that the nominal model is not perfect, i.e. $G \neq G_0$. Let

$$\Delta G(z) = G_0(z) - G(z)$$

represent the plant/model mismatch (or the model uncertainty). The closed-loop equations resulting from (A-1) and (A-5) are

$$T(z) \quad Y(z) = G_0(z)\, F(z)\, Y_d(z) + \qquad \text{(A-12)}$$

$$+ G_-(z)\,(1 - G_+(z)\, F(z)) \sum_{i=1}^{r} G_{di}(z)\, D_i(z)$$

$$T(z)\, U(z) = F(z)\, Y_d(z) - F(z) \sum_{i=1}^{r} G_{di}(z)\, D_i(z)$$

where

$$T(z) = G_-(z) + \Delta G(z)\, F(z) \qquad \text{(A-13)}$$

The closed-loop system is stable if all zeroes of T(z) lie inside the unit circle.

It is shown in Garcia and Morari (1985) that it is possible to find a filter F(z) depending on some uncertainty bounds that stabilizes the closed-loop system. A simpler way for proving this follows.

Rewrite (A-13) as

$$T(z) = G_{-}(z)\, T_1(z)$$

where

$$T_1(z) = 1 + \frac{\Delta G(z)}{G_{-}(z)}\, F(z) \qquad \text{(A-14)}$$

Since $G_{-}(z)$ is invertible (by construction) all of its zeroes are inside the unit circle. The prove stability it remains to show that the zeroes of $T_1(z)$ also lie in the unit circle.

Define the relative uncertainty as

$$\overline{\Delta G}(z) = \frac{\Delta G(z)}{G(z)} \qquad \text{(A-15)}$$

Then, from (A-14) we obtain

$$T_1(z) = 1 + \overline{\Delta G}\ (z)\ G_{+}(z)\ F(z) \quad \text{or}$$

$$T_1(z) = 1 + \overline{\Delta G}(z)\ F_{+}(z) \qquad \text{(A-16)}$$

The Nyquist criterion for stability is satisfied if

$$|\overline{\Delta G}(z)|\,|F_{+}(z)| < 1 \quad \text{for} \quad z = e^{j\omega} \quad 0 \leq \omega \leq \Pi$$

Assume that an upper bound is known for $\Delta G(z)$, i.e.

$$|\overline{\Delta G}(z)| \leq \bar{\rho}(\omega) \qquad \text{at} \quad z = e^{j\omega} \tag{A-17}$$

Then, a sufficient condition for stability is

$$|F_+(z)| < \frac{1}{\bar{\rho}(\omega)} \qquad \text{at } z = e^{j\omega}$$

or using (A-11)

$$|F_2(z)| = \left|\frac{1-f}{1-fz^{-1}}\right| < \frac{1}{\bar{\rho}(\omega)} \quad \text{at } z = e^{j\omega} \tag{A-18}$$

It is seen from (A-18) that if uncertainty bounds are known it is possible to find an f that satisfies (A-18).

Here, it will be shown formally that such an f exists if a mild condition for the steady-state gain is satisfied.

Theorem A.1

If the steady-state gain sign is known, i.e. $G_0(1)\,G(1) > 0$, there exists f_{min} $(0 \leq f_{min} < 1)$ such that for any f $(0 < f < 1)$ the closed-loop system is stable.

Proof. Substitute $z = e^{j\omega} = \cos\omega + j\sin\omega$ in (A-10). Then

$$|F_2(e^{j\omega})| = \frac{1-f}{(1+f^2-2f\cos\omega)^{1/2}}$$

and the stability condition (A-18) becomes

$$\frac{1-f}{(1+f^2-2f\cos\omega)^{1/2}} < \frac{1}{\bar{\rho}(\omega)} \qquad \text{(A-19)}$$

which is equivalent to

$$F(f;\omega) = (1+f^2-2f\cos\omega) - \bar{\rho}^2(\omega)(1-f)^2 > 0 \qquad 0 \leq \omega \leq \Pi \qquad \text{(A-20)}$$

Consider $F(f;\omega)$ as a binomial in f with ω being a parameter taking values in $[0,\pi]$.

Note that

$$F(1;\omega) = 2(1-\cos\omega) > 0 \qquad \forall \quad \omega \ (0,\pi] \qquad \text{(A-21)}$$

Let $f^*(\omega)$ be the maximum real root of $F(f^*;\omega) = 0$ in $[0,1)$. If no such root exists take $f^*(\omega) = 0$. Then

$$F(f;\omega) > 0 \qquad \forall \quad f \quad f^*(\omega) < f < 1$$

If we choose

$$f_{min} = \max_{\omega} [f^*(\omega)] \qquad 0 < \omega \leq \Pi$$

then

$$F(f;\omega) > 0 \qquad \forall \quad f \quad f_{min} < f < 1$$

$$\forall \quad \omega \quad 0 < \omega \leq \pi$$

It remains to show that (A-20) is true also for $\omega = 0$.

From (A-20)

$$F(f;0) = (1-f)^2 \, (1-\bar{\rho}^2(0)) > 0$$

which is true if $\rho(0) < 1$. Condition $\rho(0) < 1$ implies that $0 < G_0(1)/G(1) < 2$ which may be very restrictive in many cases. If the system is linear, $\rho(0) < 1$ is a reasonable assumption (usually $\rho(0) \simeq 0$, no steady-state error). But if the system is non-linear the steadiy-state gain is varying with the operating conditions. In this case the condition $\rho(0) < 1$ can be easily violated.

In the following we shall try to remove this condition. Recall that the sufficient condition for robust stability is

$$|\overline{\Delta G}(z)|| \, F_+(z)| < 1 \quad \text{for} \quad Z = e^{j\omega} \qquad 0 \leq \omega \leq \Pi$$

Imposing this condition we restrict the Nyquist diagram of $\Delta G(z)\ F_+(z)$ inside a circle of radius one.

This way we ensure that for any frequency ω the point $(-1, j_0)$ is not encircled. The condition's conservativeness can be removed at the zero frequency limit if we ensure that the Nyquist diagram starts for $\omega = 0$ at a point to the right of (-1, j0), i.e.

$$\overline{\Delta G}(1)\ F_+(1) > -1 \quad \text{or}$$

$$\frac{G_0(1)}{G(1)} > 0$$

Then (A-20) is valid for any f belonging to $(f_{min}, 1)$, q.e.d.

Asymptotic Propery of IMC

If the closed loop system is stable and the inputs are asymptotically constant (set-point and disturbances), IMC eliminates offset. This is obvious from (A-12) and the fact that $F_+(1) = 1$.

APPENDIX B
RECURSIVE LEAST SQUARES ESTIMATOR

The Least Squares Estimator

Consider the following description of a dynamical system:

$$y(t) = x^T(t-1)\theta + \xi(t-1) \tag{B-1}$$

where $y(t)$ is a measured variable, $x(t-1)$ is the information vector and is known, θ is a set of unknown parameters and $\xi(t-1)$ is an unknown sequence describing deviations from the linear deterministic dynamic behavior.

The least squares algorithm minimizes the following cost function:

$$J_t(\theta) = \sum_{i=1}^{t} \sigma_{i,t} [y(i) - x^T(i-1)\theta]^2 + (\theta-\hat{\theta}_0)^T P_0^{-1} (\theta-\hat{\theta}_0) \tag{B-2}$$

where

$$\sigma_{i,t} = \lambda(t)\, \sigma_{i,t-1} \qquad \lambda_{min} \leq \lambda(t) \leq 1 \tag{B-3}$$

$$\sigma_{i,i} = a(i) \qquad 0 \leq a(i) < \infty$$

is a sequence of nonnegative weighing coefficients, $\hat{\theta}_0$ and P_0 are apriori parameter estimates and the corresponding covariance matrix.

The estimate $\hat{\theta}(t)$ minimizing (B-2) at each time t is given by the following recursive algorithm.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + a(t)\, K(t-1)\, \varepsilon(t) \tag{B-4.1}$$

$$K(t-1) = \frac{P(t-1)\, x(t-1)}{\lambda(t) + a(t)\, w(t-1)} \tag{B-4.2}$$

$$\varepsilon(t) = y(t) - \hat{y}(t) \tag{B-4.3}$$

$$\hat{y}(t) = x^T(t-1)\, \hat{\theta}(t-1) \tag{B-4.4}$$

$$P(t) = \frac{1}{\lambda(t)} \left[P(t-1) - \frac{a(t)\, P(t-1)\, x(t-1)\, x^T(t-1)\, P(t-1)}{\lambda(t) + a(t)\, w(t-1)}\right] \tag{B-4.5}$$

$$w(t-1) = x^T(t-1)\, P(t-1)\, x(t-1) \tag{B-4.6}$$

where $P(t)$ is the covariance matrix and $\varepsilon(t)$ is the apriori prediciton error.

The algorithm is initialized with $\hat{\theta}(0) = \hat{\theta}_0$ and $P(0) = P_0$. The forgetting factor used in this work is described in section B.3.

Estimator Properties

Estimator Convergence

Define the Lyapunov function

$$V(t) = \tilde{\theta}^T(t)\, P^{-1}(t)\, \tilde{\theta}(t) \geq 0 \tag{B-5}$$

where

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t) \tag{B-6}$$

It can be proved from equations (B-4) that V(t) satisfies

$$V(t) = \lambda(t)\, V(t-1) - a(t)\, \frac{\lambda(t)\, \varepsilon^2(t)}{\lambda(t) + a(t)\, w(t-1)} + a(t)\, \xi^2(t-1) \tag{B-7}$$

and since $\lambda(t) \leq 1$,

$$V(t) \leq V(t-1) - a(t)\, \frac{\lambda(t)\, \varepsilon^2(t)}{\lambda(t) + a(t)\, w(t-1)} + a(t)\, \xi^2(t-1) \tag{B-8}$$

Summing up both sides of (B-8)

$$V(t) \leq V_0 - \sum_{i=1}^{t} \frac{a(i)\, \lambda(i)\, \varepsilon^2(i)}{\lambda(i) + a(i)\, w(i-1)} + \sum_{i=1}^{t} a(i)\, \xi^2(i-1)$$

or

$$\sum_{i=1}^{t} \frac{a(i)\, \lambda(i)\, \varepsilon^2(i)}{\lambda(i) + a(i)\, w(i-1)} \leq \sum_{i=1}^{t} a(i)\, \xi^2(i-1) + V_0 \tag{B-9}$$

Equation (B-9) is used later to derive most of the estimator properties.

Covariance Convergence

It can be proved (Leal and Goodwin, 1984) that if

$$\lim_{t \to \infty} \lambda(t) = 1 \tag{B-10}$$

the covariance is bounded

$$||P(t)|| < \infty \tag{B-11}$$

converges

$$\lim_{t\to\infty} P(t) = P_\infty \tag{B-12}$$

and satisfies

$$\lim_{t\to\infty} \frac{a(t)\ P(t-1)\ x(t-1)\ x^T(t-1)\ P(t-1)}{\lambda(t) + a(t)\ w(t-1)} = 0 \tag{B-13}$$

Given these general estimator properties more useful ones are generated when specific forms of $\xi(t)$ are considered.

Linear Deterministic Systems Case, $\xi(t) = 0$

For these systems we take $a(t) = 1$. It is obvious from (B-2) that

$$\lim_{t\to\infty} \frac{\varepsilon^2(t)}{\lambda(t) + w(t-1)} = 0 \tag{B-14}$$

Then it is easy to show from (B-4) and (B-14) the following:

i. $|\hat{\theta}(t)| < M \qquad \forall\ t$

ii. $|\hat{\theta}(t) - \hat{\theta}(t-i)| \to 0 \qquad \forall$ finite i

iii. $|\varepsilon(t)| \leq \delta_1(t) + \delta_2(t)\ |x(t-1)|$

If, also,

$$\lim_{t\to\infty} \lambda(t) = 1$$

we have

$$\text{iv. } \lim_{t\to\infty} P(t) = P_\infty \qquad \text{(B-15)}$$

$$\text{v. } \lim_{t\to\infty} \hat{\theta}(t) = \hat{\theta}_\infty$$

Properties (i)-(iii) are well known and can be found in any standard textbook for adaptive control (e.g. Goodwin and Sin, 1984, p. 69). Proof for (iv) nad (v) can be found in Leal and Goodwin (1984). Property (v) means that the estimated parameters converge but not necessarily to the true ones.

Bounded Disturbance Csae

Consider that $\xi(t)$ is a bounded sequence such that

$$\sup |\xi(t)| \leq \Delta$$

In this case, the following choice for a(t)

$$a(t) = \begin{cases} 1 & \text{if } \dfrac{\lambda_{min}\, \varepsilon^2(t)}{1 + w(t-1)} > \Delta^2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(B-16)}$$

allow us to prove similar properties for the estimator as in the $\xi(t) = 0$ case.

These properties are summarized in the following Lemma B.1. The proof is included since Lemma B.1 has not been proved in the literature for the estimator described in (B-4).

Lemma B.1

Consider the system given by (B-1) and the estimator described by (B-4) and (B-16). Further, assume that

$$\sup |\xi(t)| < \Delta$$

and

$$\lim_{t\to\infty} \lambda(t) = 1$$

Then the estimator has the following properties:

i. $|\hat{\theta}(t)| < M_1 < \infty$

ii. $$\frac{\varepsilon^2(t)}{\lambda(t) + w(t-1)} < M_2 < 0$$

iii. $\lim_{t\to\infty} |\tilde{\theta}(t) - \tilde{\theta}(t-i)| = 0 \qquad i < M$

iv. $|\varepsilon(t)| \leq M_3 + \delta(t)\,|x(t-1)|$

where $0 \leq \delta(t) < \infty$ and $\lim \delta(t) = 0$

and $0 < M_3 < \infty$

Also

$|\mu(t)| < M_3$

where

$\mu(t) = y(t) - x^T(t-1)\,\hat{\theta}(t)$

is the aposteriori prediction error.

v. $\lim_{t\to\infty} a(t)\left[\dfrac{\lambda_{min}\,\varepsilon^2(t)}{1 + w(t-1)} - \Delta^2\right] = 0$

Proof.

i. It is obvious from (B-8) and (B-16) that the Liapunov function V(t) satisfies

$$V(t) - V(t-1) \leq 0 \qquad \text{(B-18)}$$

Since $\lim_{t\to\infty} \lambda(t) = 1$, P(t) is bounded. Then (B-8) and (B-5) imply that $|\hat{\theta}(t)|$ is bounded. The algorithm modification (B-16) is the key to the proof here. It was specifically designed to make (B-18) possible.

ii. If a(t) = 0, then (B-16) ensures that

$$\frac{\varepsilon^2(t)}{\lambda(t) + w(t-1)} \leq \frac{1 + w(t-1)}{\lambda_{min} + w(t-1)}\,\frac{\Delta^2}{\lambda_{min}} < M_2$$

Otherwise, a(t) = 1 and from (B-8) we obtain

$$\frac{\lambda(t)\,\varepsilon^2(t)}{\lambda(t) + w(t-1)} \leq V(t-1) - V(t) + \Delta^2$$

From (B-18)

$$V(t-1) - V(t) \leq V_0 - V(t) \leq V_0$$

so

$$\frac{\varepsilon^2(t)}{\lambda(t) + w(t-1)} \leq \frac{V_0 + \Delta^2}{\lambda_{min}} < M_2$$

q.e.d.

iii. From (B-4.1)

$$|\tilde{\theta}(t) - \tilde{\theta}(t-1)|^2 = a^2(t)\ |k(t-1)|^2\ s(t-1)\ \frac{\varepsilon^2(t)}{s(t-1)} \tag{B-19}$$

where $s(t-1) = \lambda(t) + w(t-1)$. From (B-13) and (B-4.2) follows that

$$\lim_{t \to \infty} a(t)\ |k(t-1)|^2\ s(t-1) = 0$$

and property (iii) follows easily from (B-19) and property (ii).

iv. From (B-4.3) and (B-4.4)

$$\varepsilon(t) = x^T(t-1)\ \tilde{\theta}(t-1) + \xi(t-1) \tag{B-20}$$

and similarly

$$\mu(t) = x^T(t-1)\ \tilde{\theta}(t) + \xi(t-1) \tag{B-21}$$

Subtracting θ from both sides of (B-4.1) and premultiplying by $x^T(t-1)$ we obtain

$$\mu(t) = \varepsilon(t) + a(t)\ x^T(t-1)\ k(t-1)\ \varepsilon(t)$$

and using (B-4.2)

$$\mu(t) = \frac{\lambda(t)\ \varepsilon(t)}{\lambda(t) + a(t)\ w(t-1)}$$

which in view of (ii) gives

$$|\mu(t)| \leq M_3 = \left(\frac{M_2}{\lambda_{min}}\right)^{1/2} \qquad \text{(B-22)}$$

Subtracting (5-21) from (5-20) we have

$$\varepsilon(t) = \mu(t) + [\tilde{\theta}(t-1) - \tilde{\theta}(t)]^T\ x(t-1)$$

which in view of (iii) and (B-22) results to

$$|\varepsilon(t)| \leq M_3 + \delta(t)\ |x(t-1|$$

where

$$\lim_{t\to\infty} \delta(t) = \lim_{t\to\infty} |\tilde{\theta}(t-1) - \tilde{\theta}(t)| = 0$$

v. From (B-9)

$$\sum_{i=1}^{t} a(i) \left[\frac{\lambda_{min}\ \varepsilon^2(t)}{1 + w(i-1)} - \Delta^2\right] \leq V_0$$

which implies

$$\lim_{t\to\infty} a(t) \left[\frac{\lambda_{min}\ \varepsilon^2(t)}{1 + w(t-1)} - \Delta^2\right]_{]} = 0$$

Asymptotically Zeroing Disturbances

Assume $\xi(t)$ is a bounded sequence which satisfies

$$\sum_{i=0}^{\infty} \xi^2(i) < M \tag{B-23}$$

and

$$\lim_{t \to \infty} \xi(t) = 0 \tag{B-24}$$

Then, from (B-9) we obtain

$$\lim_{t \to \infty} \frac{\epsilon^2(t)}{\lambda(t) + w(t-1)} = 0$$

and the same properties as in the deterministic case are valid.

Forgetting Factor

Following Fortescue et al. (1981), the forgetting factor $\lambda(t)$ is given by:

$$\lambda(t) = \max \{\bar{\lambda}(t), \lambda_{min}\}$$

and

$$\bar{\lambda}(t) = 1 - \frac{a(t)}{S_0} \left[\frac{\epsilon^2(t)}{1 + w(t-1)} - \frac{\Delta^2}{\lambda_{min}} \right] \tag{B-25}$$

where S_0 is a positive constant.

It is seen from property (v) of Lemma B.1 that

$$\lim_{t \to \infty} \lambda(t) = 1 \tag{B-26}$$

which is required for proving properties (i)-(iv) in Lemma B.1. Note that the only requirement for $\lambda(t)$ in the proof of (v) in Lemma B.1 is

$$\lambda_{min} \leq \lambda(t) < 1$$

Modified Recursive Least Squares Estimator for Adaptive MDC

The usual least squares estimator given by (B-4) is employed. This estimator satisfies the properties E1-E3, E6 stated in section 6.4.

Pointwise Stabilizability (Leal and Goodwin, 1984)

Let $\hat{N}(t)$ represent the Sylvester eliminant of (6-18). Then, if

$$|\det (\hat{N}(t))| < \delta \qquad \delta > 0$$

modify the parameters using

$$\bar{\theta}(t) = \hat{\theta}(t) + P(t)\, z(t) \tag{B-27}$$

where $z(t)$ is a bounded vector, and form $\bar{N}(t)$, the new Sylvester eliminant based on $\bar{\theta}(t)$. A $z(t)$ that satisfies

$$|\det (\bar{N}(t))| \geq \delta$$

can be found by a finite trial and error search.

Constrained Least Squares Algorithm (Goodwin and Sin, 1984, p. 92)

If

$$|\hat{B}(1) < b_{min}$$

then project the estimates on the hyperplane

$$B(1) = \text{sign}\,(B(1))\; b_{min} \tag{B-28}$$

This is achieved using (B-27) where z(t) is given as described in Goodwin and Sin (1984, p. 94).

It is shown in Leal and Goodwin (1984) and in Goodwin and Sin (1984) that the above modifications do not affect the other properties of the parameter estimator.

APPENDIX C
PROPERTIES OF MDC POLE PLACEMENT ALGORITHM

Consider the MDC pole placement Bezutian equation

$$T = AQ + BP \tag{C-1}$$

where A, B, T are knovn polynomials, and Q, P are found as the solution of (C-1).

Let

$$A(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}$$

$$B(z) = z^{-k} (b_0 + b_1 z^{-1} + \ldots + b_1 z^{-1})$$

$$T(z;r) = a_0 + r a_1 z^{-1} + r^2 a_2 z^{-2} + \ldots + r^n a_n z^{-n}$$

The minimal solution of (C-1) is obtained if

$$\deg(Q) = n_q = l + k - 1$$

$$\deg(P) = n_p = n - 1$$

Then, let

$$Q(z) = q_0 + q_1 z^{-1} + \ldots + q_{n_p} z^{-n_p}$$

$$P(z) = p_0 + p_1 z^{-1} + \ldots + p_{n_p} z^{-n_p}$$

and define

$$w_1 = [q_0\ q_1 \cdots q_{n_p}\ p_0\ p_1 \cdots p_{n_p}]^T \qquad [(n+1+k) \times 1]$$

$$\tau_1(r) = [a_0,\ r\,a_1,\ \ldots,\ r^n a_n,\ 0 \ldots 0]^T \qquad [(n+1+k) \times 1]$$

The solution of the Bezoutian (C-1) is given by the system of linear equations

$$\tau_1 = M\, w_1 \tag{C-2}$$

where M is the Sylvester Eliminant matrix for the polynomials A(z), B(z). Its entries are the coefficients of A(z), B(z).

Noting that the leading coefficients of A(z) and T(z) are the same and $k \geq 1$, we can show easily that $q_0 = 1$. Then (Goodwin and Sin, 1984, p. 482) we can rewrite (C-2) as

$$\tau = a + Nw \tag{C-3}$$

where

$$w = [q_1\ q_2\ \cdots\ q_{n_p}\ p_0\ p_1\ \cdots\ p_{n_p}]^T$$

$$a = [a_1\ a_2\ \cdots\ a_n\ 0\ \cdots\ 0]^T \qquad [(n+l+k-1)\ x\ 1]$$

$$\tau = \Lambda(r)\ a = [\ ra_1,\ r^2 a_2,\ \ldots,\ r^n a_n,\ 0\ \cdots\ 0]^T$$

(C-4)

$$\Lambda(r) = \begin{bmatrix} r & & & & \\ & r^2 & & & \\ & & \ddots & & \\ & & & r^n & \\ & & & & 0 \\ & & & & & \ddots \\ & & & & & & 0 \end{bmatrix}$$

and N is obtained from M if we omit the first row and column.

From (C-3) the unknown vector w is found from

$$w(r) = -\ N^{-1}\ [I - \Lambda(r)]\ a \qquad (C-5)$$

Note that we can write

$$I - \Lambda(r) = (1 - r)\ \Lambda_1(r)$$

and substituting back to (C-5) we obtain

$$w(r) = - (1 - r) N^{-1} \Lambda_1(r) a \qquad (C-5)$$

In view of (C-5) we can express P(z) and Q(z) as

$$P(z) = (1 - r) \bar{P}(z) \qquad (C-6)$$

$$Q(z) = 1 + (1 - r) \bar{Q}(z) \qquad (C-7)$$

Equation (C-6) is useful in proving the robustness of MDC law.

APPENDIX D
PROPERTIES OF TIME VARYING LINEAR DISCRETE OPERATORS

Consider the time varying operator polynomials in the backwards shift operator q^{-1},

$$A(t,q^{-1}) = a_0(t) + a_1(t)\, q^{-1} + \ldots + a_n(t)\, q^{-n}$$

$$B(t,q^{-1}) = b_0(t) + b_1(t)\, q^{-1} + \ldots + b_m(t)\, q^{-m}$$

$$C(t,q^{-1}) = c_0(t) + c_1(t)\, q^{-1} + \ldots + c_l(t)\, q^{-l}$$

Define the operator product

$$A(t,q^{-1})\, B(t,q^{-1}) = \sum_{i}^{n} \sum_{j}^{m} a_i(t)\, b_j(t)\, q^{-i-j} \qquad \text{(D-1)}$$

and the operator convolution

$$A(t,q^{-1}) \cdot B(t,q^{-1}) = \sum_{i}^{n} \sum_{j}^{m} a_i(t)\, b_j(t-i)\, q^{-i-j} \qquad \text{(D-2)}$$

Let $y(t)$ be a time sequence and define the operator-sequence product as

$$A(t,q^{-1})\, y(t) = \sum_{i=0}^{n} a_i(t)\, y(t-i)$$

The operator product and convolution satisfy the properties given in Lemma D.1 (henceforth q^{-1} is suppressed).

Lemma D.1

Given the linear discrete time varying operators A(t), B(t), C(t) and a time sequence y(t), the following are true:

i. The operator product

a. is commutative

$$A(t)\ B(t) = B(t)\ A(t) \tag{D-3}$$

b. satisfies

$$[A(t)\ B(t)]\ y(t) \neq A(t)\ [B(t)\ y(t)] \tag{D-4}$$

c. is distributive with respect to addition

$$A(t)\ [B(t) + C(t)] = A(t)\ B(t) + A(t)\ C(t) \tag{D-5}$$

ii. the operator convolution

a. is non-commutative

$$A(t) \cdot B(t) \neq B(t) \cdot A(t) \tag{D-6}$$

b. is associative

$$[A(t) \cdot B(t)]\ y(t) = A(t)\ [B(t)\ y(t)] \tag{D-7}$$

c. is distributive with respect to addition

$$A(t) \cdot [B(t) + C(t)] = A(t) \cdot B(t) + A(t) \cdot C(t) \tag{D-8}$$

iii. Satisfies

$$\Delta AB(t) \equiv A(t)\ B(t) - A(t) \cdot B(t)$$

$$= \sum_{i}^{n} \sum_{j}^{m} a_i(t)\ [b_j(t) - b_j(t-i)]\ q^{-i-j} \tag{D-9}$$

iv. $$A(t) \cdot B(t) = B(t) \cdot A(t) + \Delta BA(t) - \Delta AB(t) \tag{D-10}$$

Proof.

i. a,b are obvious from definition (D-1) and c follows from the fact that the operators are linear.

ii. a. It is obvious from definition (D-2).

b. From (D-2)

$$[A(t) \cdot B(t)]\, y(t) = \sum_i^n \sum_j^m a_i(t)\, b_j(t-i)\, y(t-i-j)$$

$$= \sum_i^n a_i(t)\, q^{-i} \sum_j^m b_j(t)\, y(t-j)$$

$$= A(t)\, [B(t)\, y(t)]$$

c. Follows easily from (D-2) and operator linearity.

iii. From definition (D-9) using (D-1) and (D-2) we have

$$\Delta AB(t) = \sum_i^n \sum_j^m a_i(t)\, b_j(t)\, q^{-i-j} - \sum_i^n \sum_j^m a_i(t)\, b_j(t-i)\, q^{-i-j}$$

$$= \sum_i^n \sum_j^m a_i(t)\, [b_j(t) - b_j(t-i)]\, q^{-i-j}$$

iv. From (D-9)

$$A(t)\, B(t) = A(t) \cdot B(t) + \Delta AB(t)$$

and

$$B(t)\ A(t) = B(t) \cdot A(t) + \Delta BA(t)$$

and (D-10) follows in view of (D-3).

The properties of Lemma D.1 are used in the following to prove Lemma 6.1, for the adaptive MDC law.

Lemmas 6.1 - Proof

Recall that in the non-adaptive case the closed-loop equations are given by (5-9) and (5-10). Similarly, consider here the linear time varying operator $\hat{T}_c(t)$ defined from

$$\hat{T}_c(t) = \hat{A}(t)\ \hat{H}(t) + \hat{B}'(t)\ \hat{K}(t) \qquad \text{(D-11)}$$

It is easy to show using (6-14) to (6-16) that

$$\hat{T}_c(t) = \hat{B}_-(t)\ \hat{F}_D(t)\ \hat{T}(t) + \tilde{B}(t)\ \hat{K}(t) \qquad \text{(D-12)}$$

where

$$\tilde{B}(t) = \hat{B}'(t) - \hat{B}(t) \qquad \text{(D-13)}$$

We shall try to derive a closed-loop model for the system of (6-1) and (6-13) which is based on (D-11).

We proceed first for the system output. From (D-11) we obtain

$$\hat{T}_c(t)\, y(t) = [\hat{A}(t)\ \hat{H}(t)]\, y(t) + [\hat{B}'(t)\ \hat{K}(t)]\, y(t) \tag{D-14}$$

which in view of (D-3), (D-9), and (D-7) becomes

$$\hat{T}_c(t)\, y(t) = \hat{H}(t)\ [\hat{A}(t)\, y(t)] + \hat{B}(t)\ [\hat{K}(t)\, y(t)] + \hat{\Delta H} \hat{A}(t)\, y(t) + \hat{\Delta B}' \hat{K}(t) \tag{D-15}$$

It is easy to prove from (6-5)-(6-7) and (6-40) that

$$\hat{A}(t)\, y(t) = \hat{B}(t)\, u(t) + \hat{d}(t) + \eta(t) \tag{D-16}$$

which if substituted in (D-15), gives

$$\hat{T}_c(t)y(t) = \hat{H}(t)[\hat{B}'(t)u(t)] + \hat{B}'(t)[\hat{K}(t)y(t)] + \hat{H}(t)\hat{d}(t) + \hat{H}(t)\eta(t) + \hat{\Delta H} \hat{A}(t)y(t) + \hat{\Delta B}' \hat{K}(t)y(t) \tag{D-17}$$

Using (D-7) and (D-10) to reverse $\hat{H}(t) \cdot \hat{B}'(t)$ we obtain

$$\hat{T}_c(t)y(t) = \hat{B}'(t)\ [\hat{H}(t)u(t) + \hat{K}(t)y(t)] + \hat{H}(t)\hat{d}(t) + \hat{H}(t)\eta t) + \hat{\Delta H} \hat{A}(t)y(t) + \hat{\Delta B}' \hat{K}(t)y(t) + \hat{\Delta B}' \hat{H}(t)u(t) - \hat{\Delta H} \hat{B}'(t)u(t)$$

Finally, from (6-12) and the above we obtain

$$\hat{T}_c(t)y(t) = \hat{B}'(t)\ [\hat{R}(t)y_{sp}(t)] + \hat{H}(t)\hat{d}(t) + \hat{H}(t)\eta(t) + \qquad \text{(D-18)}$$

$$+ [\hat{\Delta H}\hat{A}(t) + \hat{\Delta B}'\hat{K}(t)]\ y(t) + [\hat{\Delta B}'\hat{H}(t) - \hat{\Delta H}\hat{B}'(t)]\ u(t)$$

The closed-loop equation for the control variable is obtained in a similar fashion staring from

$$\hat{T}_c(t)\ u(t) = [\hat{A}(t)\ \hat{H}(t)]\ u(t) + [\hat{B}'(t)\ \hat{K}(t)]\ u(t)$$

and is given by

$$\hat{T}_c(t)\ u(t) = \hat{A}(t)\ [\hat{R}(t)\ y_{sp}(t)] - \hat{K}(t)\ \hat{d}(t) - \hat{K}(t)\ \eta(t) \qquad \text{(D-19)}$$

$$+ [\hat{\Delta A}\hat{K}(t) - \hat{\Delta K}\hat{A}(t)]\ y(t) + [\hat{\Delta A}\hat{K}(t) + \hat{\Delta K}\hat{B}'(t)]\ u(t)$$

Combining (D-18), D(19), and (D-22) Lemma 6.1 is obtained.

Next a Lemma is presented concerning properties of slowly time varying operators. They are needed in proving Lemma D.2 and Lemmas 6.2, and 6.4.

Definition 1. We say that the operator $A(t,q^{-1})$ is bounded iff

$$|a_i(t)| < \infty \qquad \forall\ t \qquad i = 0, \ldots, n$$

Definition 2. We say that the operator $A(t,q^{-1})$ is slowly time varying iff

$$\lim_{t\to\infty} |a_i(t+1) - a_i(t)| = 0 \qquad i = 0, \ldots, n$$

Lemma D.2

Consider the time varying operators $A(t,q^{-1})$, $B(t,q^{-1})$, $C(t,q^{-1})$. Then, the following are true:

i. Suppose that both $A(t,q^{-1})$ and $B(t,q^{-1})$ are bounded and slowly time varying. Then the operators

$$C(t,q^{-1}) = A(t,q^{-1})\, B(t,q^{-1}) \tag{D-20}$$

$$D(t,q^{-1}) = A(t,q^{-1}) + B(t,q^{-1}) \tag{D-21}$$

are also bounded and slowly time varying.

ii. Suppose that $C(t,q^{-1})$ with leading coefficient

$$|c_0(t)| \geq \varepsilon > 0$$

is bounded and slowly time varying. Then the monic polynomials $A(t,q^{-1})$ and $B(t,q^{-1})$ satisfying

$$C(t,q^{-1}) = c_0(t)\, A(t,q^{-1})\, B(t,q^{-1}) \qquad \text{(D-22)}$$

are bounded and slowly time varying.

Proof.

i. Since $A(t,q^{-1})$, $B(t,q^{-1})$ are bounded and slowly time varying, it is true from definitions 1 and 2 that

$$\lim_{t\to\infty} |a_i(t+1) - a_i(t)| = 0 \qquad |a_i(t)| < \infty \qquad \text{(D-23)}$$

$$\lim_{t\to\infty} |b_i(t+1) - b_i(t)| = 0 \qquad |b_i(t)| < \infty \qquad \text{(D-24)}$$

From (D-20) the coefficients $c_i(t)$ of $C(t,q^{-1})$ are given by

$$c_i(t) = \sum_{k=0}^{i} a_k(t)\, b_{i-k}(t) \qquad \text{(D-25)}$$

In view of (D-23) and (D-24) it follows

$$|c_i(t)| < \infty$$

From (D-25) we obtain

$$c_i(t+1) - c_i(t) = \sum_{k=0}^{i} [a_k(t+1)\, b_{i-k}(t+1) - a_k(t)\, b_{i-k}(t)]$$

or

$$|c_i(t+1) - c_i(t)| \leq \sum_{k=0}^{i} [|b_{i-k}(t+1) - b_{i-k}(t)||a_k(t)| + |a_k(t+1) - a_k(t)||b_{i-k}(t+1)|]$$

and using (D-23) and (D-24)

$$\lim_{t\to\infty} |c_i(t+1) - c_i(t)| = 0$$

Thus, $C(t,q^{-1})$ is bounded and slowly time varying.
From (D-21)

$$d_i(t) = a_i(t) + b_i(t)$$

It is trivial to show that

$$|d_i(t)| < \infty$$

and

$$\lim_{t\to\infty} |d_i(t+1) - d_i(t)| = 0$$

ii. Consider (D-22) as a factorization of $C(t,q^{-1})$ and let $z_i(t)$ represent its i^{th} zero. We shall show first that all zeroes of $C(t,q^{-1})$ are bounded and slowly time varying. Any $z_i(t)$ satisfies

$$c_0(t)\, z_i^{l}(t) + c_1(t)\, z_i^{l-1}(t) + \ldots + c_l(t) = 0 \qquad \text{(D-26)}$$

Since $|c_0(t)| \geq \varepsilon > 0$ $|z_i(t)$ is bounded. Similarly to (D-26) at time t + k we have

$$c_0(t+k)\, z_i^{l}(t+k) + c_1(t+k)\, z_i^{l-1}(t+k) + \ldots + c_l(t+k) = 0 \qquad \text{(D-27)}$$

Subtracting (D-26) for (D-27) and factoring we obtain

$$[z_i(t+k) - z_i(t)]\ \{\text{other terms} \neq 0\} = (c_0(t+k) - c_0(t))\, z_i^{l}(t+k) + \ldots + (c_l(t+k) - c_l(t)) \qquad \text{(D-28)}$$

Since $C(t,q^{-1})$ is slowly time varying and $z_i(t+k)$ is bounded, it follows from (D-28) that

$$\lim_{t\to\infty} |z_i(t+k) - z_i(t)| = 0 \qquad \text{(D-29)}$$

Now, $A(t,q^{-1})$ and $B(t,q^{-1})$ can be expressed as products with factors $(1-z_i(t)q^{-1})$ which are bounded and slowly time varying. Then, the first part of Lemma D.2 implies that both $A(t,q^{-1})$ and $B(t,q^{-1})$ are bounded and slowly time varying.

Lemmas concerning the properties of the adaptive MDC time varying polynomials are presented next.

Lemma D.3

If the estimator properties E1-E6 are valid then the following are true:

i. $|\beta(t)| \geq \beta_{min} > 0$

ii. the adaptive MDC law polynomials $\hat{Q}(t,q^{-1})$, $\hat{P}(t,q^{-1})$ and $\hat{K}(t,q^{-1})$ are bounded

iii. there exists $\varepsilon_{1,max} > 0$ such that the polynomial $T_c(t,q^{-1})$ which is obtained by taking $\varepsilon_1 = \varepsilon_{1,max}$ is stable for every time t

iv. the MDC law polynomials $\hat{B}_-(t)$, $\hat{B}_+(t)$, $\hat{\bar{B}}_+(t)$, $\hat{C}(t)$, $\hat{T}(t)$, $\hat{P}(t)$, $\hat{Q}_1(t)$ are all bounded and slowly time varying

v. the MDC law operators $\hat{H}(t)$, $\hat{K}(t)$, $\hat{R}(t)$ are bounded and slowly time varying.

Proof.

i. From the estimator property E5

$$|\hat{B}'(1)| \geq b_{min} \tag{D-30}$$

From (6-29) and (6-34) we obtain

$$|\hat{B}'(1)| \leq |\hat{B}(1)| + \tilde{l}\,\varepsilon_1$$

Using (D-30) and step 0 of the algorithm

$$b_{min}\left(1 - \frac{\tilde{l}}{l}\right) < |\hat{B}(1)|$$

In view of (6-9)-(6-12), the above equation implies that

$$|\beta(t)| > b_{min} \left(1 - \frac{\tilde{l}}{l}\right) \prod_{i=1}^{l_1(t)} \frac{1}{|1-z_i^-|} \prod_{i=1}^{l_1(t)} \frac{1}{|1-1/z_i^+|}$$

or

$$|\beta(t)| \geq \beta_{min} > 0$$

ii. Consider first the Bezoutian equation (6-18)

$$\hat{T}(t) = \hat{C}(t)\, \hat{Q}_1(t) + \hat{B}_+(t)\, \hat{P}(t)$$

It is proved in Appendix C that (6-18) is equivalent to (equation (C-5))

$$w(t) = - N^{-1}(t) [I - \Lambda(r(t))]\, a(t) \qquad \text{(D-31)}$$

where w(t) is a vector with entries the coefficients of $\hat{Q}_1(t)$ and $\hat{P}(t)$, a(t) is a vector containing the coefficients of C(t), and N(t) is the Sylvester Eliminant of (6-18).

From (D-31)

$$|w(t)| \leq \frac{||N(t)||^{N-1}}{|\det (N(t))|} (1 - r(t)^{n_c})\, |a(t)| \; ; \quad N = \deg (N(t)) \qquad \text{(D-32)}$$

From E1 and Remark 6.1 follows that $\hat{C}(t)$ and $\hat{B}_+(t)$ are bounded, and consequently $||N(t)||$ and $|a(t)|$ are bounded. In addition, property E4 implies that $|\det (N(t))|$ is bounded away from zero. Then, it follows from (D-32) that $|w(t)|$ is bounded and the same is true for $\hat{Q}_1(t)$ and $\hat{P}(t)$. Then, part (ii) of Lemma D.3 follows easily from (6-19).

iii. Recall that a sufficient condition for stability of $T_c(t)$ is given by (6-36). In view of part (ii) of this Lemma $\hat{d}_{max}$ is bounded. Next note that step 1 and step 2 of the adaptive MDC algorithm ensure that all zeroes $\hat{B}_-(t)$ $\hat{T}_-(t)$ are inside a circle of radius $1-\sigma$ in the complex plane. From the latter and part (i) of this Lemma it follows that d_{min} is bounded away from zero. Then (6-36) implies that there exists $\varepsilon_{1,max} > 0$ so that for $\varepsilon_1 = \varepsilon_{1,max}$ (6-36) is satisfied for all times.

iv. The estimator property E6 implies that for any $\delta > 0$ there exists a finite time t_0 such that

$$|\hat{b}_i(t) - \hat{b}_i(t_0)| < \delta(t_0) \qquad \forall \ t \geq t_0 \tag{D-33}$$

Part (iii) of this Lemma and (D-33) imply that if there are $\hat{b}_i$'s that converge to values smaller than $\varepsilon_{1,max}$, then there exist a finite line t_0 such that

$$|\hat{b}_j(t)| < \varepsilon_{1,max} \qquad \forall \ t \geq t_0$$

and $\hat{b}_j(t)$ is not included in $\hat{B}(t,q^{-1})$ for $t \geq t_0$. This is equivalent to $k(t) = k$ and $m(t) = m$ for $t \geq t_0$. Now from (D-33), (D-28), (6-26), and (6-27) follows that the m-k zeroes of $\hat{B}(t,q^{-1})$ satisfy

$$|z_i(t) - z_i(t_0)| < \delta_1(t_0) \qquad t \geq t_0 \tag{D-34}$$

which implies that the number of stable (or unstable) zeroes does not change after a finite time t_0, i.e. $l_1(t) = l_1$, $l_2(t) = l_2$, and $l_3(t) = l_3$ for $t \geq t_0$.

Now, having established the convergence of the orders of $\hat{B}(t,q^{-1})$, $\hat{B}_-(t,q^{-1})$, and $\hat{B}_+(t,q^{-1})$ in finite time, it follows easily from Lemma D.2 (ii) and the estimator property E2 that $\hat{B}_-(t)$, $\hat{B}_+(t)$, and $\hat{\bar{B}}_+(t)$ are slowly time varying polynomials. Then using Lemma D.2 (i) and (6-39) it is straight-forward to show that $\hat{C}(t)$ and $\hat{T}(t)$ are bounded and slowly time varying.

Next we shall use (D-31) to show that $P(t)$, $Q_1(t)$ are also slowly time varying. Let

$$\tau(t) = -[I - A(r(t))]\, a(t)$$

and rewrite (D-31) as

$$\tau(t) = N(t)\, w(t)$$

from which we obtain

$$w(t+1) - w(t) = N^{-1}(t)(\tau(t+1) - \tau(t)) - N^{-1}(t)(N(t+1) - N(t))w(t+1) \tag{D-35}$$

Since both $a(t)$ and $r(t)$ are slowly time varying

$$\lim_{t\to\infty} ||\tau(t+1) - \tau(t)|| = 0 \tag{D-36}$$

Similarly from property E2

$$\lim_{t\to\infty} ||N(t+1) - N(t)|| = 0 \tag{D-37}$$

In view of (D-36) and (D-37) we obtain from (D-35)

$$\lim_{t\to\infty} |w(t+1) - w(t)| = 0$$

or $\hat{Q}_1(t)$ and $\hat{P}(t)$ are slowly time varying polynomials.

v. This property follows easily in view of properties (ii)-(iv) of this Lemma and Lemma D.2.

Finally, we prove Lemma 6.4.

Lemma 6.4 Proof

From (6-41) and (6-42) we have

$$l_1(t) = \hat{B}'(t)\,[\hat{R}(t)\,y_{sp}(t)] + \hat{H}(t)\,\eta(t) + H'(t)\,\hat{d}(t)$$

$$+ [\hat{\Delta B}'\hat{K}(t) + \hat{\Delta H}\hat{A}(t)]\,y(t) + [\hat{\Delta B}'\hat{H}(t) - \hat{\Delta H}\hat{B}'(t)]\,u(t) \qquad \text{(D-38)}$$

It follows from E1-E3 and Lemma D.3 that the first three terms in the RHS of (D-37) are bounded. Since $\hat{A}(t)$, $\hat{B}'(t)$, $\hat{H}(t)$, $\hat{K}(t)$ are all slowly time varying, equation (D-9) gives

$$|[\hat{\Delta B}'\hat{H}(t) - \hat{\Delta H}\hat{B}'(t)]\,u(t)| \leq \gamma_2(t)\,||u(t)||_\infty$$

and

$$|[\hat{\Delta B}'\hat{K}(t) + \hat{\Delta H}\hat{A}(t)]\,y(t)| \leq \gamma_2(t)\,||y(t)||_\infty$$

where

$$\lim_{t\to\infty} \gamma_2(t) = 0$$

Then (6-41) follows easily and similarly (6-42).

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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